Identifying coherent homogeneous regions for extreme rainfall.

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28 January 2020

Trajectories
Univ. Grenoble Alpes

LABORATOIRE
JEAN KUNTZMANN
MATHÉMATIQUES APPLIQUÉES - INFORMATIQUE

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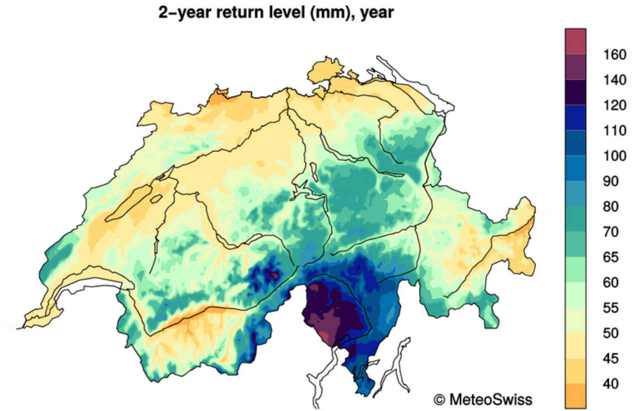
(Flash) flood

- More common natural disaster
- Caused by heavy precipitation (extreme rainfall)

Flood in SW of France

Ex : Maximum observed 412mm/day vs 900mm/year in Grenoble

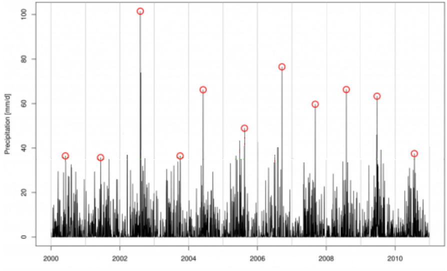
Defining coherent regions for extreme rainfall

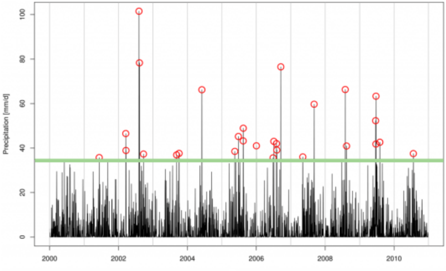


What about longer return period ?

Goal: Estimating **return levels** of unobserved events.

Two approaches: **GEV**(μ, σ, ξ) (maxima) or **GPD**(σ, ξ) (threshold)





Regional Frequency Analysis Hosking et al. [1985]

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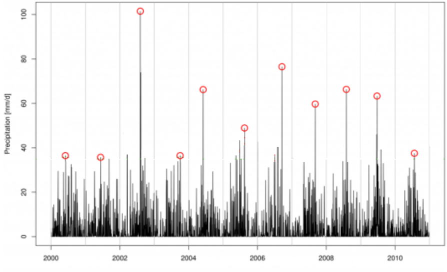

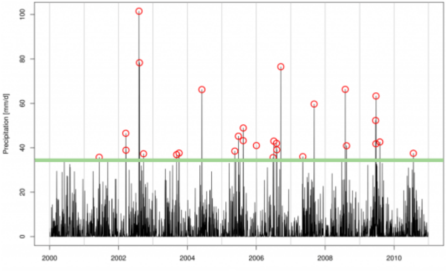

Problems: Asymptotic distributions + lack of information

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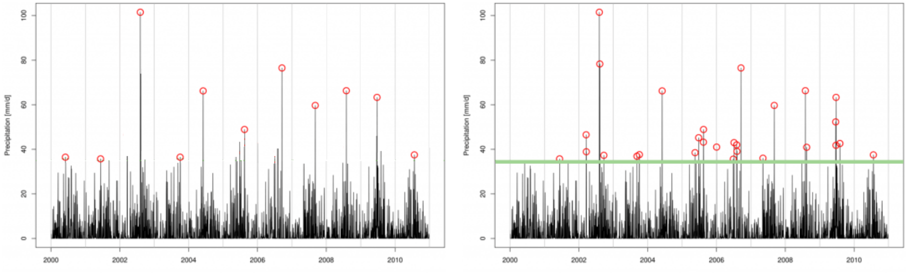

Problems: Asymptotic distributions + lack of information

Solution: Regional Frequency Analysis **(RFA)**

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- Limits

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RFA: What's that ?

Hypothesis: Region is **homogeneous** iff

$$Y_i = \sigma_i Y, \quad \sigma_i \geq 0$$

e.g $Y \sim GEV, GPD, ...$

Goal: Gathering weather stations with same distribution (except scale factor).

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Methods available: Region of Influence (**RoI**) vs partitioning.

Regions of influence (RoI) Evin et al. [2016]

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4. Homogeneous region = **RoI** (Region of Influence)

Remark

A same station can belong to several RoI.

Partitionning into subregions Carreau et al. [2017]

Principle: Gathering stations by measured precipitation.

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Remark

One region per station with this method.

Limits of these approaches

- **RoI**: Choice of the characteristics
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What's new ?

A threshold-free clustering algorithm.

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   - Threshold-free model
   - Clustering methods
4. Application to Switzerland

Objective

Identifying homogeneous regions for extreme rainfall.

Methode based on :

- **Partitionning** in homogeneous regions
- **Probability Weighted Moments** (PWM)
- Extended model (no block, no threshold) : **EGPD**

Probability Weighted Moments

Definition

Let $X \underset{c.d.f}{\sim} F$*, a random variable. PWM of order* k *of* X *is defined as*

$$\alpha_k := \mathbb{E}\left[XF(X)^k\right]$$

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Theorem (Guillou et al. [2009])

For $k = 0, 1, 2,$

$$\sqrt{n}\,(\hat{\alpha}_k - \alpha_k) \xrightarrow{d} N(0, \sigma_k^2)$$

where

$$\hat{\alpha}_k := \frac{1}{n}\sum_{i=1}^{n}\left(\frac{i}{n}\right)^k X_{(i)}$$

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Goal : Modeling low, moderate and heavy rainfall intensities (EVT).

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Theorem

The precipitation variable Y can be modeled as

$$Y = \sigma H_{\xi}^{-1}\left[G^{-1}(U)\right], \ \sigma \in \mathbb{R}_{+}^{*}$$

where $U \sim \mathcal{U}[0,1]$, H_{ξ}^{-1} inverse c.d.f of $GPD(1,\xi)$ and G a skewed c.d.f.

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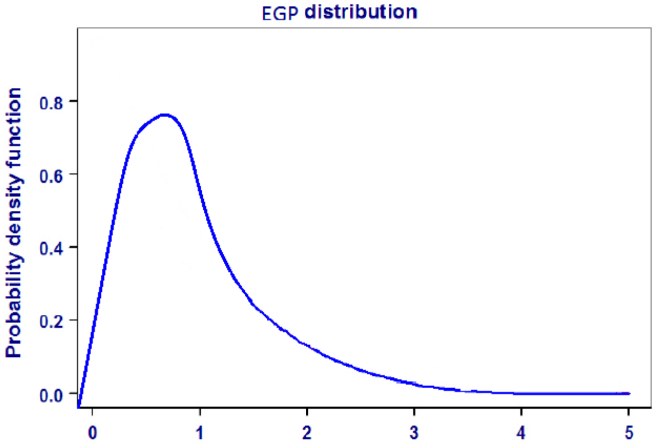
Remark

For instance, $G : u \longmapsto u^{\kappa}$ suits.

EGPD density function



EGPD p.d.f

Describing the tail distribution

Definition (Diebolt et al. [2008])

$$R(X) = \frac{3\alpha_2 - \alpha_0}{2\alpha_1 - \alpha_0}$$

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$$R(X) = \frac{3\alpha_2 - \alpha_0}{2\alpha_1 - \alpha_0}$$

Theorem (Naveau et al. [2016])

R(EGPD) only depends on κ *and* ξ,

$$R(EGPD) = \frac{3B(3\kappa, 1-\xi) - B(\kappa, 1-\xi)}{2B(2\kappa, 1-\xi) - B(\kappa, 1-\xi)}$$

Method

Reminder: We aim at partitionning space in homogeneous regions for extreme rainfall.

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3. Estimating the at-site ratio R of PWM $\rightarrow \hat{R} = \dfrac{3\hat{\alpha}_2 - \hat{\alpha}_0}{2\hat{\alpha}_1 - \hat{\alpha}_0}$

Method

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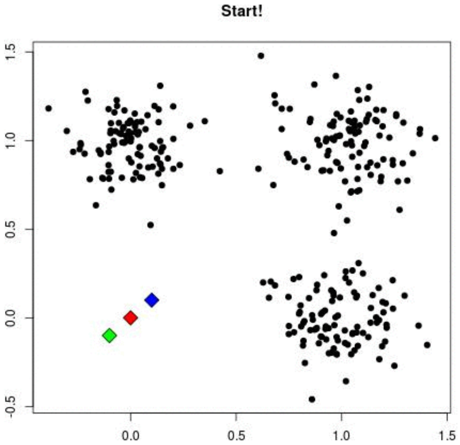
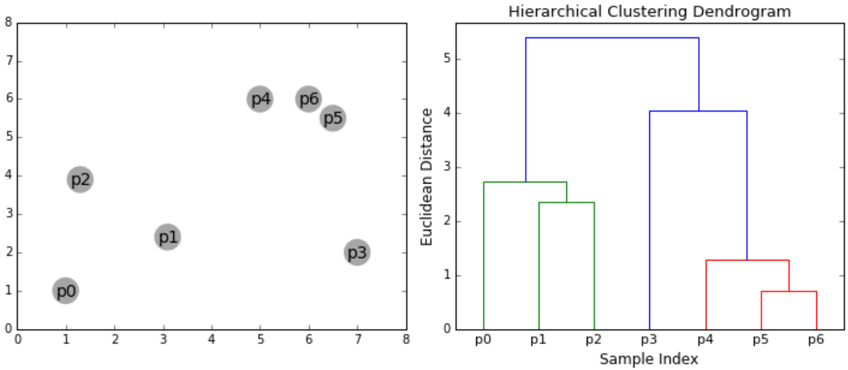
1. Sampling (all positive precipitation)
2. Estimating the at-site PWM $\rightarrow \alpha_k$
3. Estimating the at-site ratio R of PWM $\rightarrow \hat{R} = \dfrac{3\hat{\alpha}_2 - \hat{\alpha}_0}{2\hat{\alpha}_1 - \hat{\alpha}_0}$
4. Clustering the at-site ratio estimations: Hierarchical clustering, K-means or K-medoids

K-means Du et al. [2006]



Hierarchical clustering



Battle of clustering methods

	K-means/medoids	**HCA**
Drawbacks	Initialisation dependent	Costly
Advantages	Cheap	Initialisation independent

Remark

K-medoids $\sim$ with centers belonging to the dataset.

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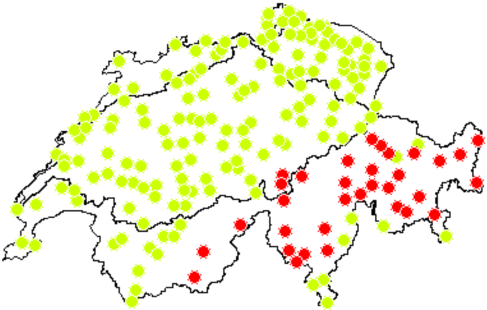
- Clustering on pair-dependence

Application to Switzerland

Dataset: Daily rainfall, 85 years.

Weather stations in Switzerland

Application to Switzerland



Clustered stations by marginal distributions

Limitations

This approach does not take into account:

- Spatial dependence
- Temporal dependence
- Non-stationarity

Spatial dependence Bador et al. [2015]

Measure of pair-synchronicity with the **F-madogram**.

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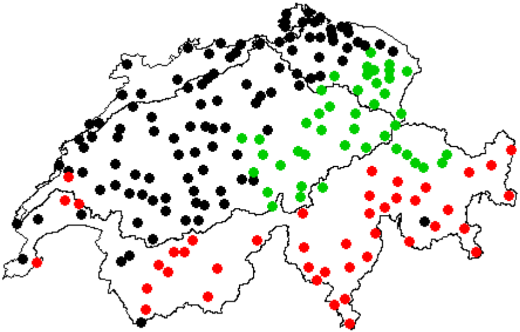

Clustered stations by spatial dependence

Wanted: ideas

How to combine clustering based on marginal distributions and pair-dependence ?

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Thank you for your attention !

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