Certificates for Triangular Equivalences and Rank Profiles

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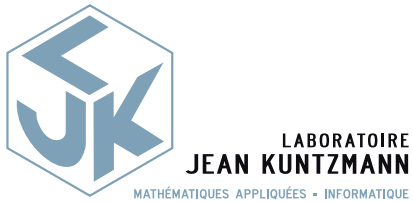


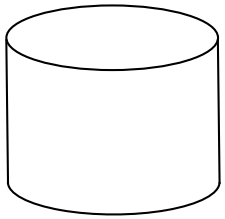
Outline

1. Introduction
2. Our certificates
3. Conclusion

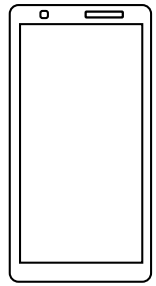
Plan

1. Introduction
2. Our certificates
3. Conclusion

Context: outsourced computation



Server



Client

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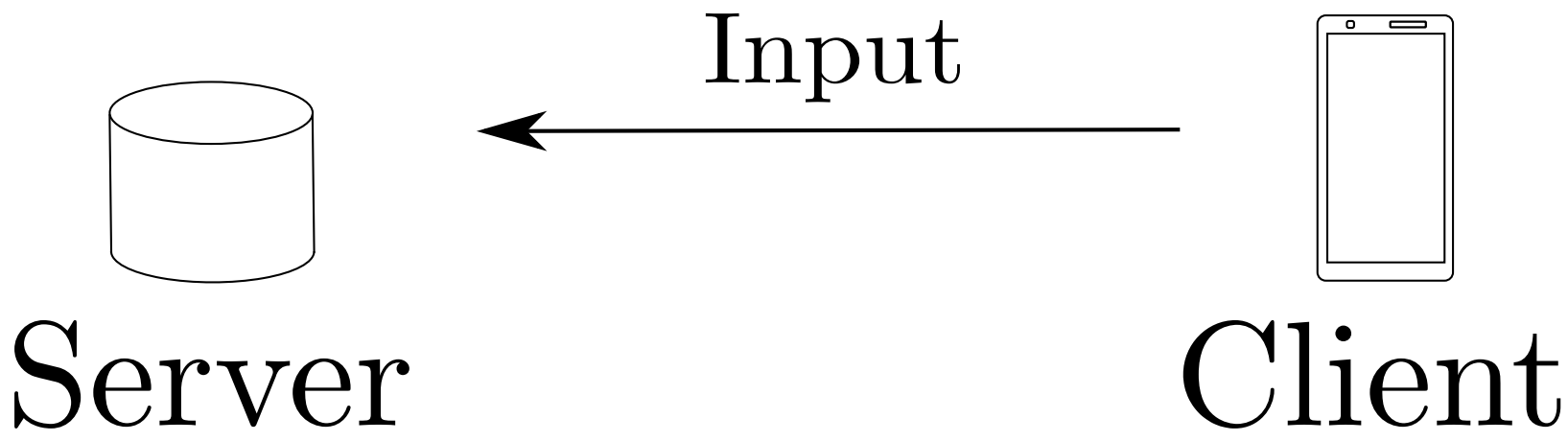

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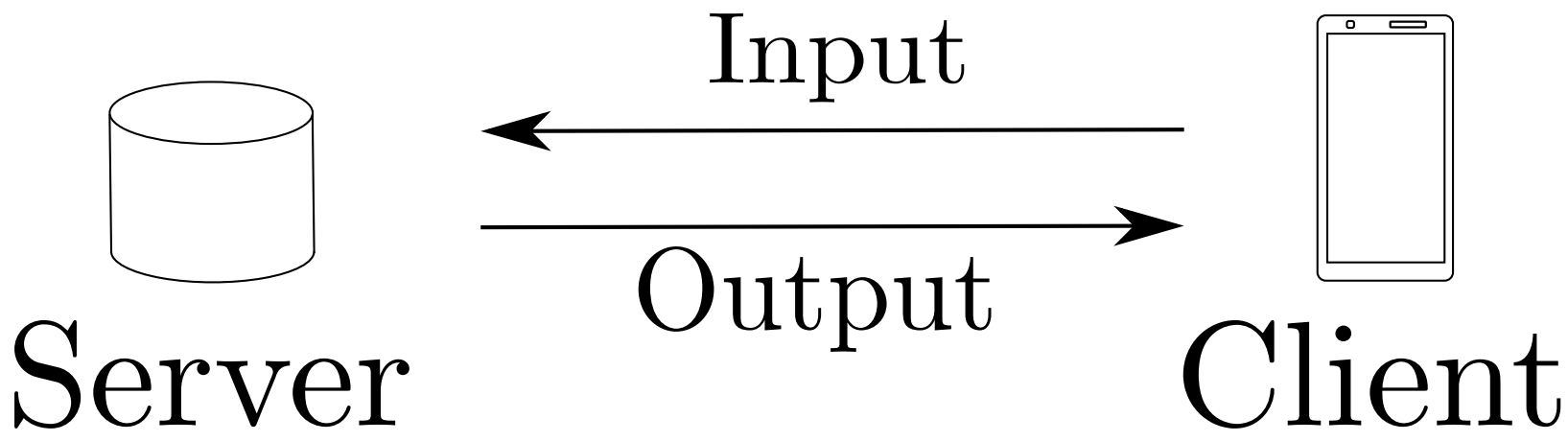

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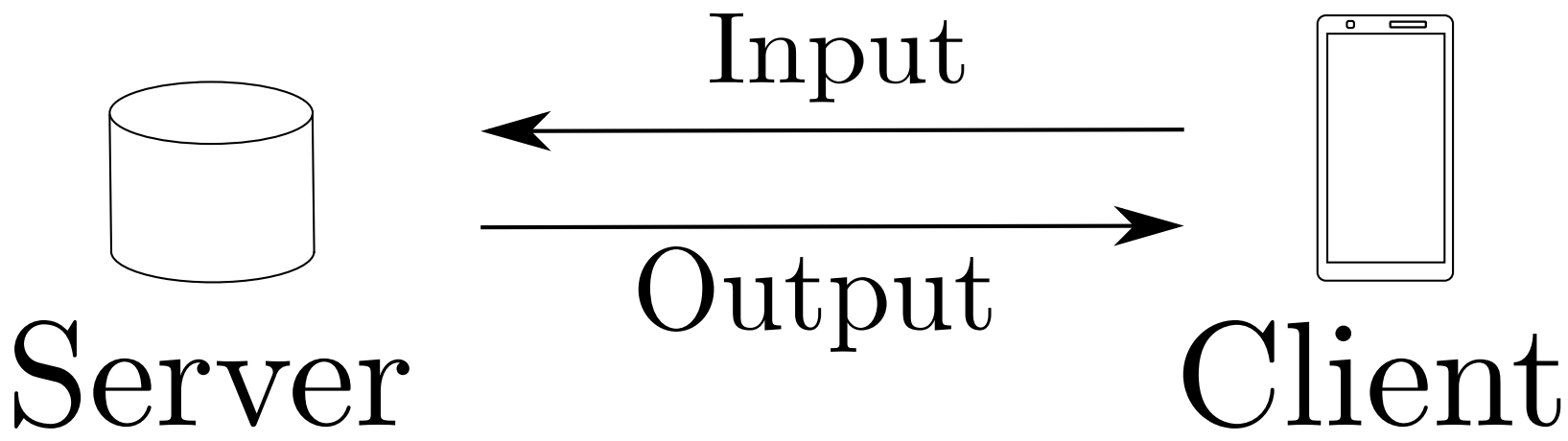

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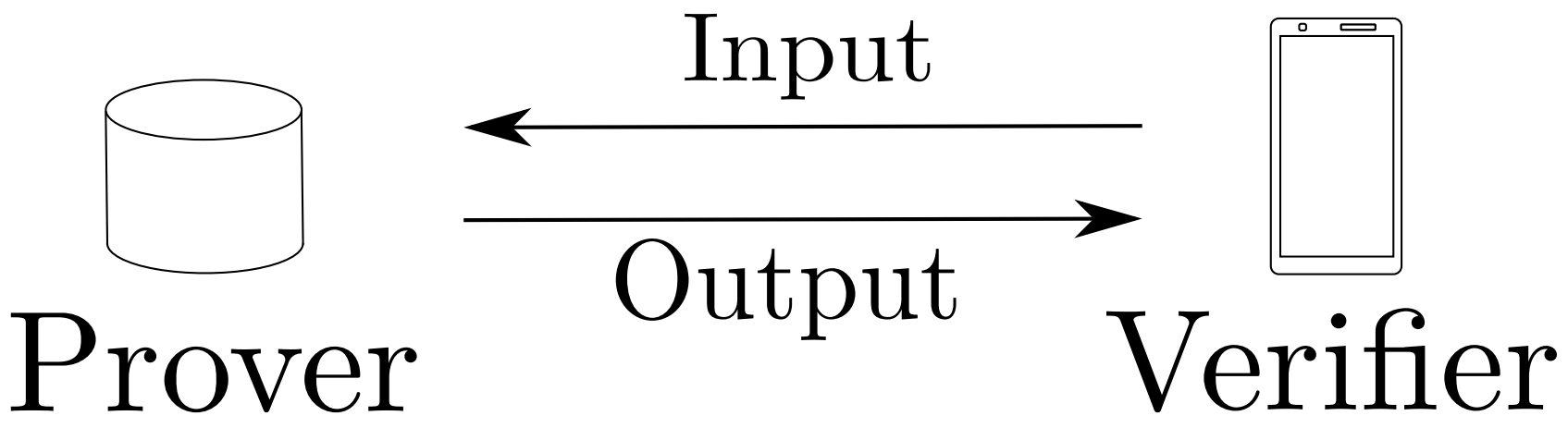

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An example: Freivalds' certificate [F79]

Prover		Verifier
	$A, B \in \mathbf{F}^{n \times n}$	
$C = AB$	$\xrightarrow{C}$	Sample a random vector v
		$Cv \stackrel{?}{=} A(Bv)$

- **Probabilistic** check for matrix product
- Complexity measures

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$O(n^\omega)$	$O(n^2)$	$O(n^2)$

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Rank profile matrix

Rank Profiles

Given a matrix A of rank r:

Example

$$A = \begin{pmatrix} 0 & 0 & 0 & 2 \\ 3 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 \end{pmatrix}$$

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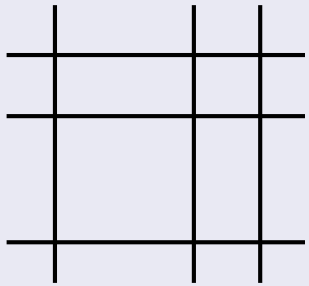

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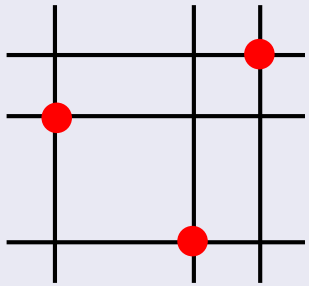

RPM (Rank Profile Matrix)

The unique $\mathcal{R}_A$ such that any pair of (i, j)-leading sub-matrix of $\mathcal{R}_A$ and of A have the same rank.

Certificates approaches

- Several approaches for certificates
- Generic strategies based on crypto/non-crypto techniques
  - Often have at least a large constant overhead
  - [GKR08], [CFHKKNPZ15], ...

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  - Soft factor on communication space and verifier time
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- A strategy for linear algebra algorithms reducing to matrix multiplication
  - Soft factor on communication space and verifier time
  - [KNS11]
- Algorithm specific approach
  - Optimize global cost

Comparison with [KNS11] approach

Certificate	Algorithm	Prover time	Comm.	Verifier time
CRP/RRP/RPM	[KNS11]+[DPS16]	Best known	$\widetilde{O}(mn)$	$\widetilde{O}(mn)$

- $A \in \mathbf{F}^{m \times n}$ of rank r
- Examples of possible Provers :
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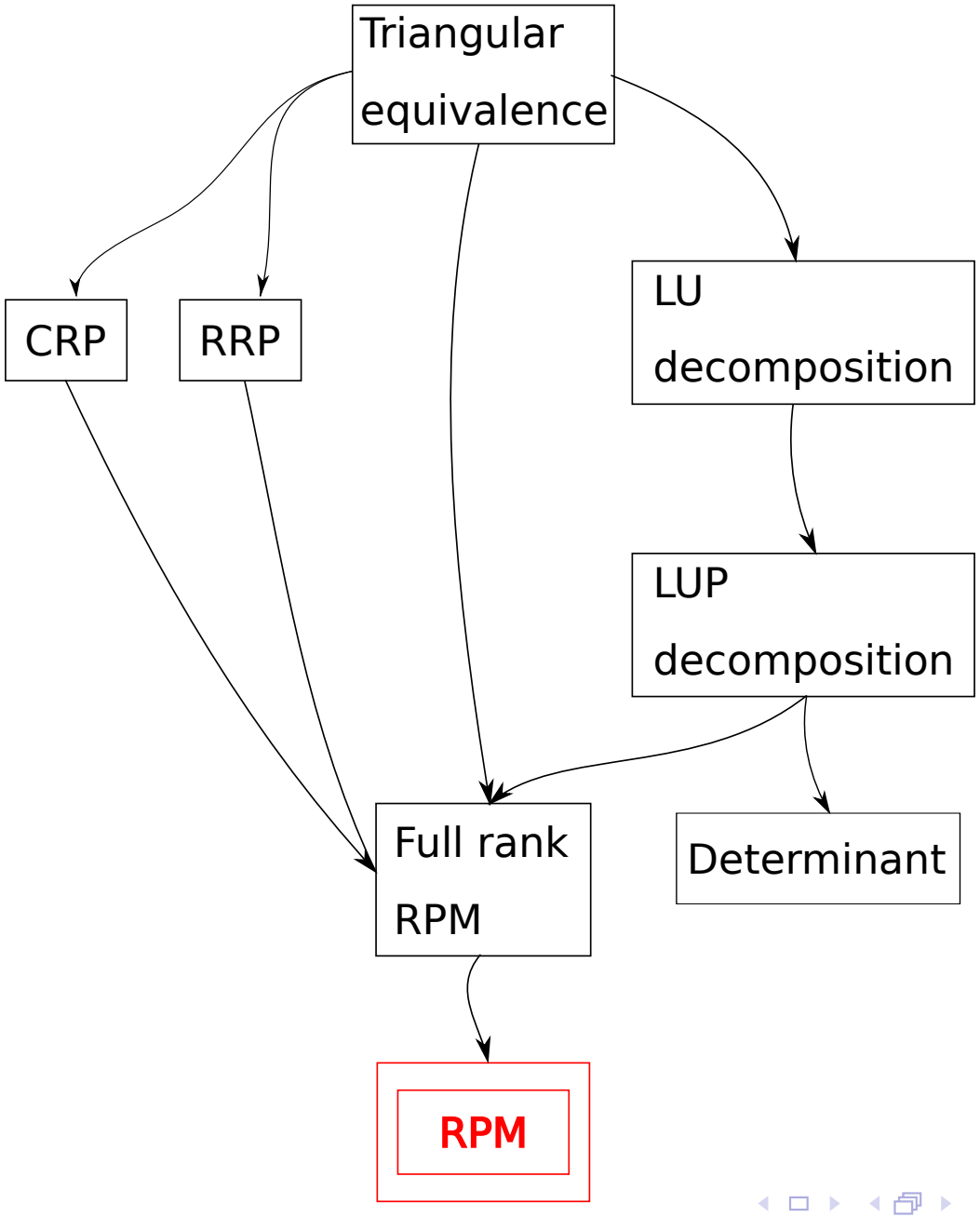
Triangular equivalence
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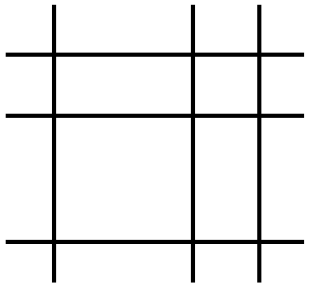
3: Extract the submatrix 

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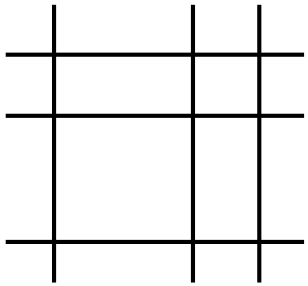
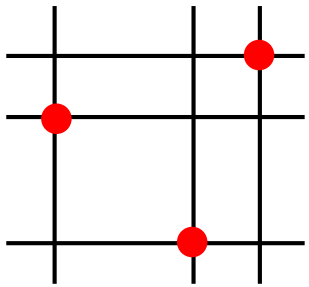
3: Extract the submatrix 
4: Compute the Rank Profile Matrix of the submatrix of A 

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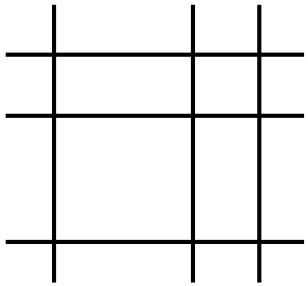
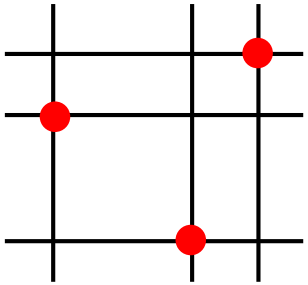
3: Extract the submatrix 
4: Verify the Rank Profile Matrix of the submatrix of A 

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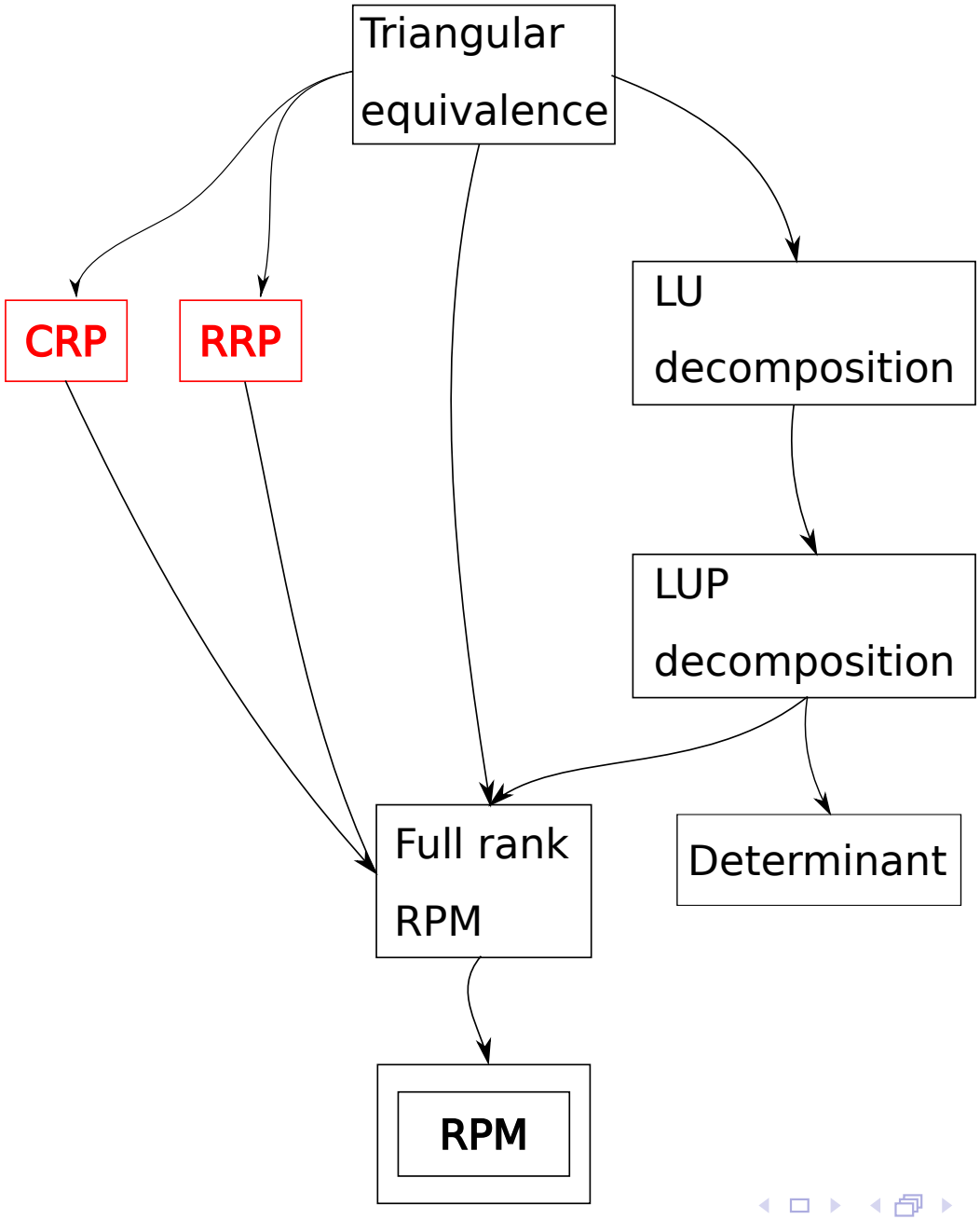
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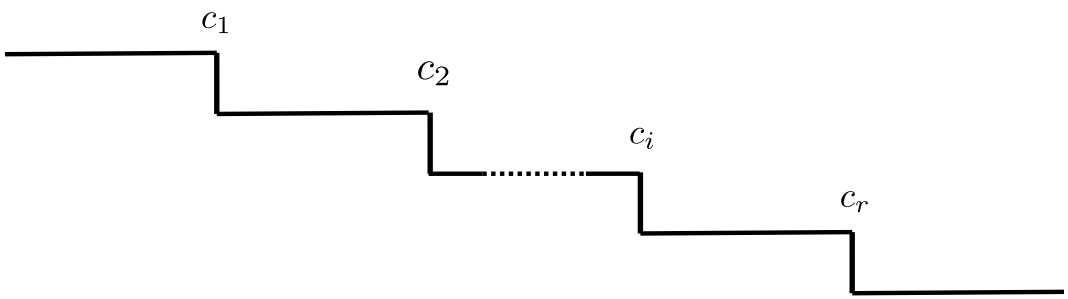

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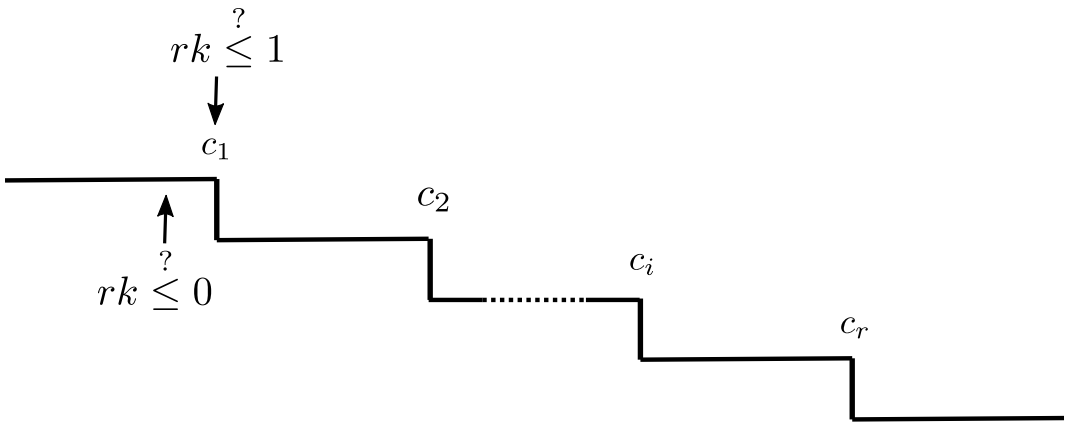

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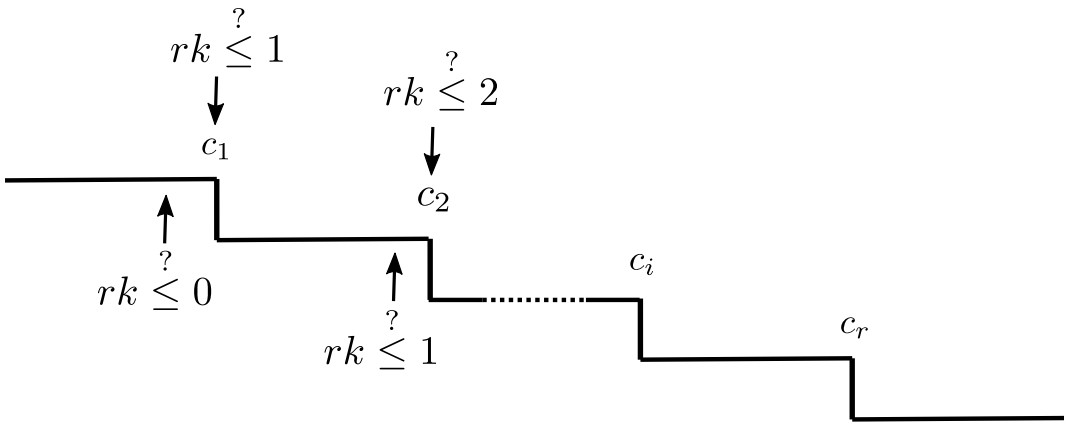

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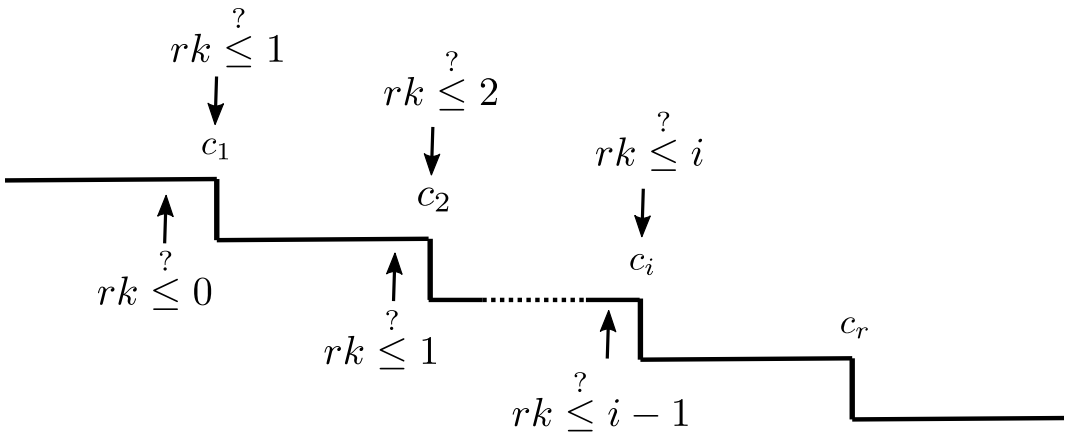

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- Use it to verify the column rank profile?
- Problem: $2rO(n)$ comm.

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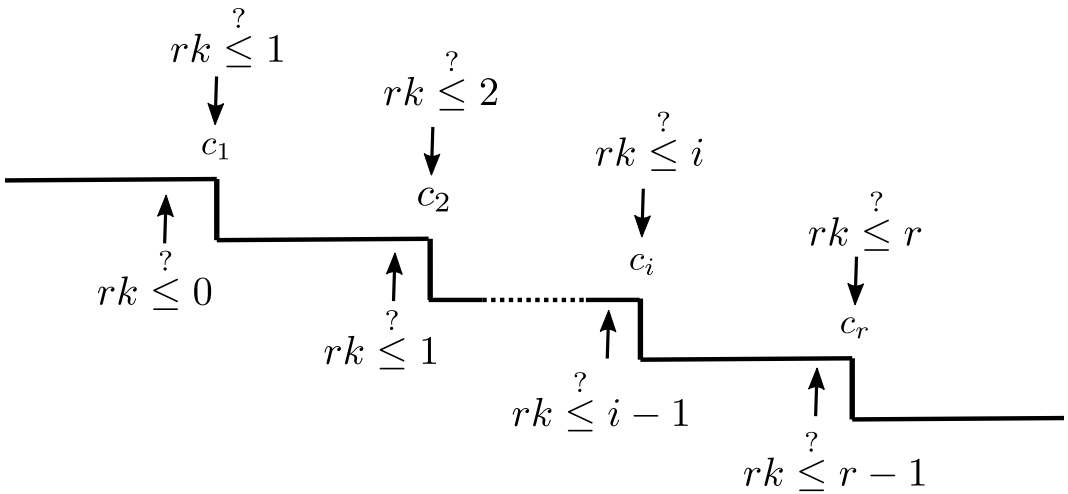

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- Still too much comm. space

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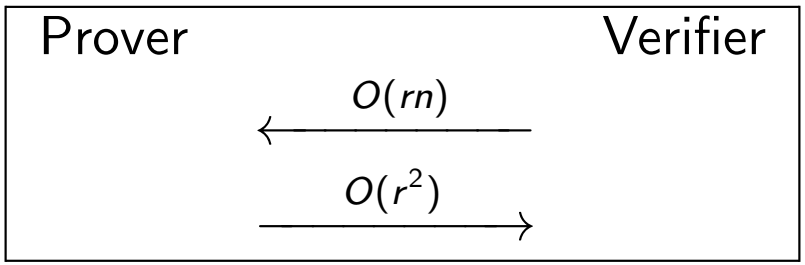

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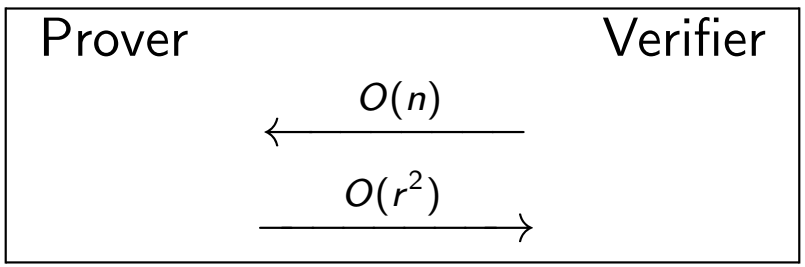

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- Use vectors with the same prefix for all challenges
- Still quadratic for comm. space...
- Verify a projection of Γ

$\rightarrow$ Triangular equivalence

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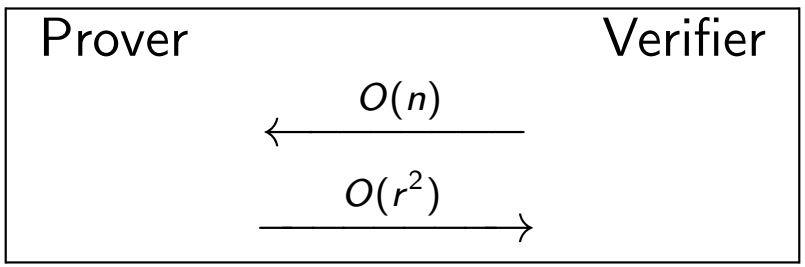
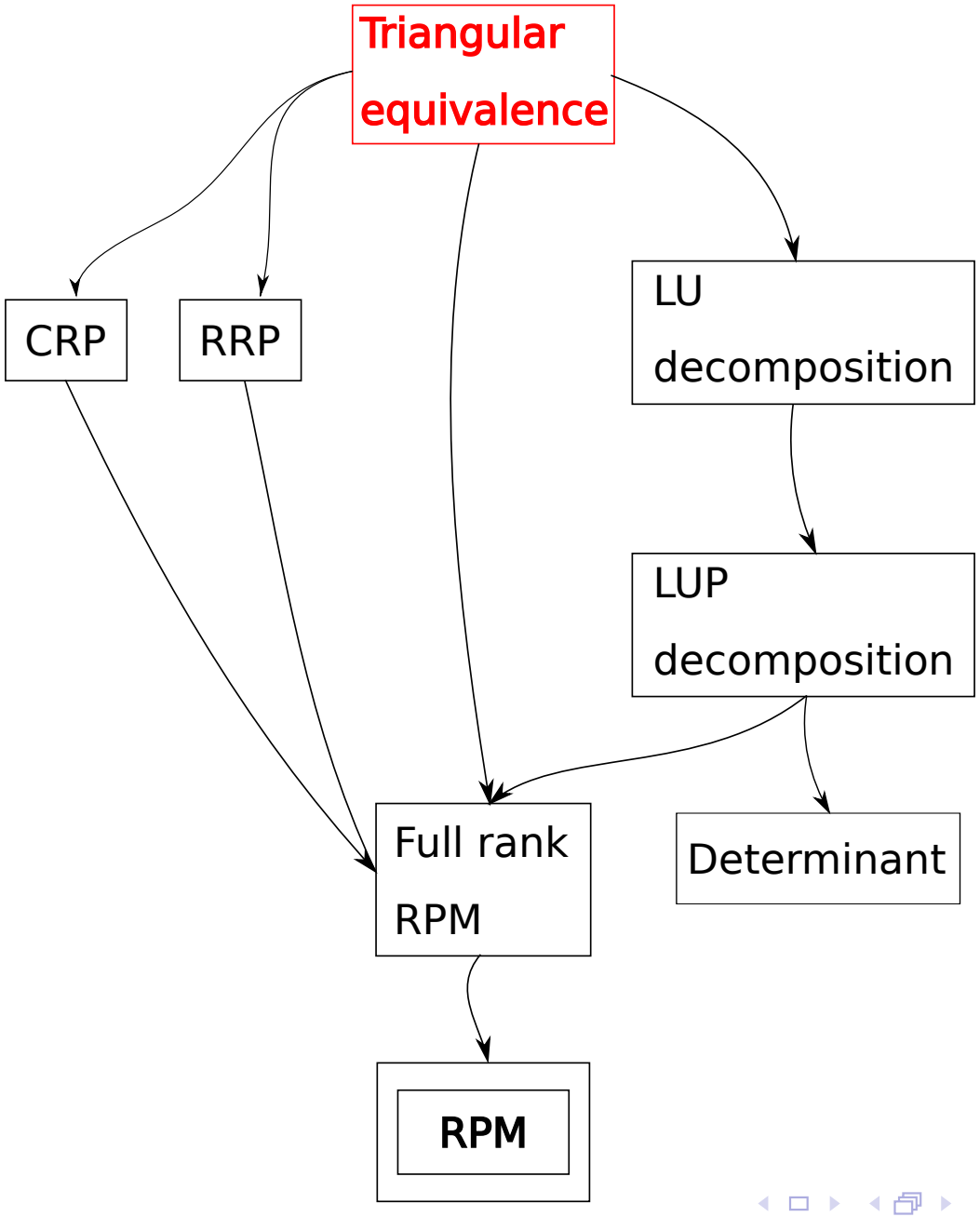


Roadmap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Triangular equivalence

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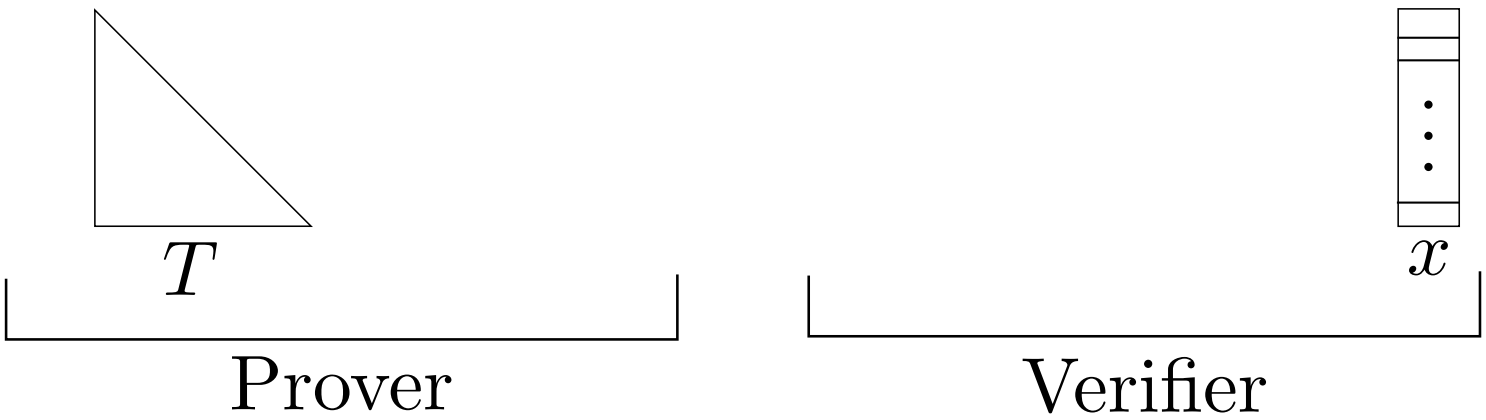

Triangular equivalence

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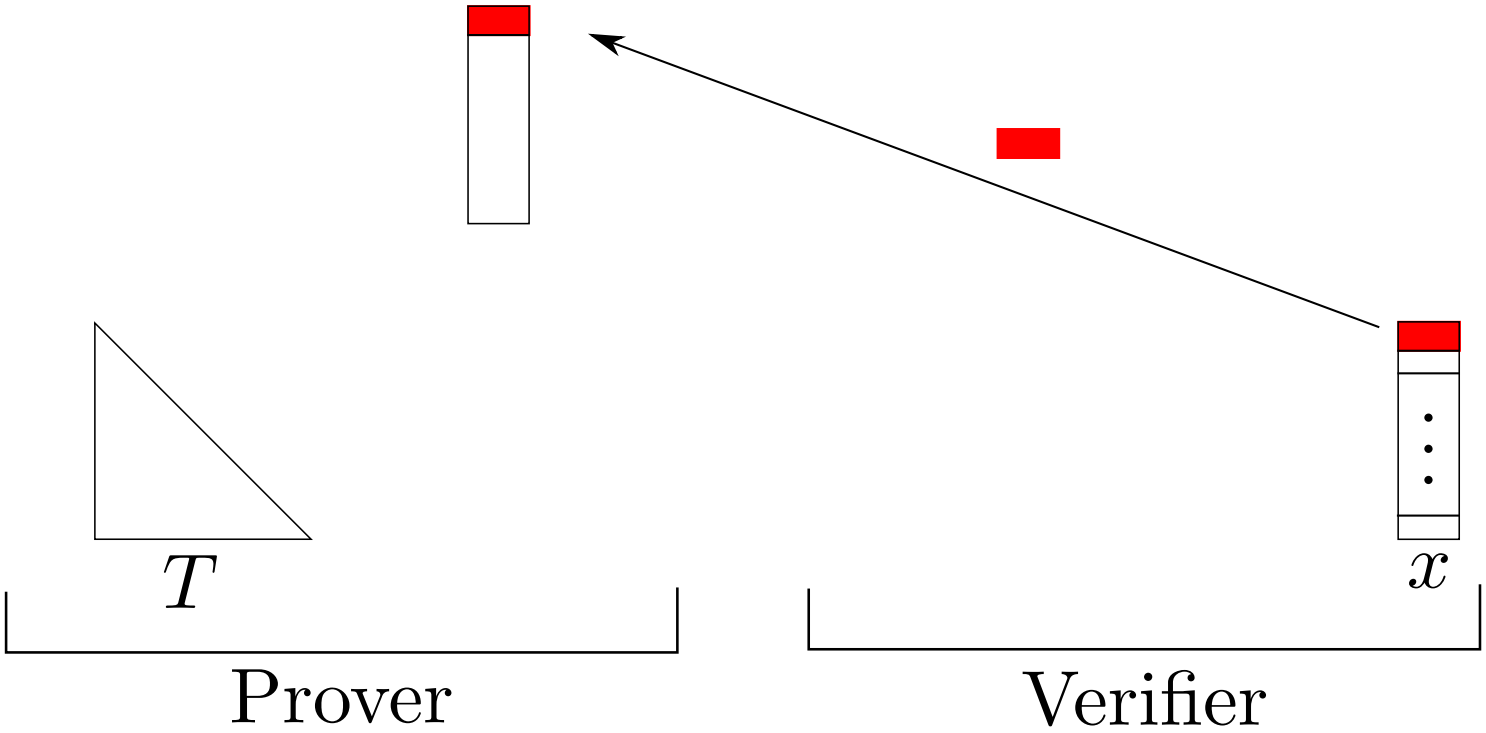

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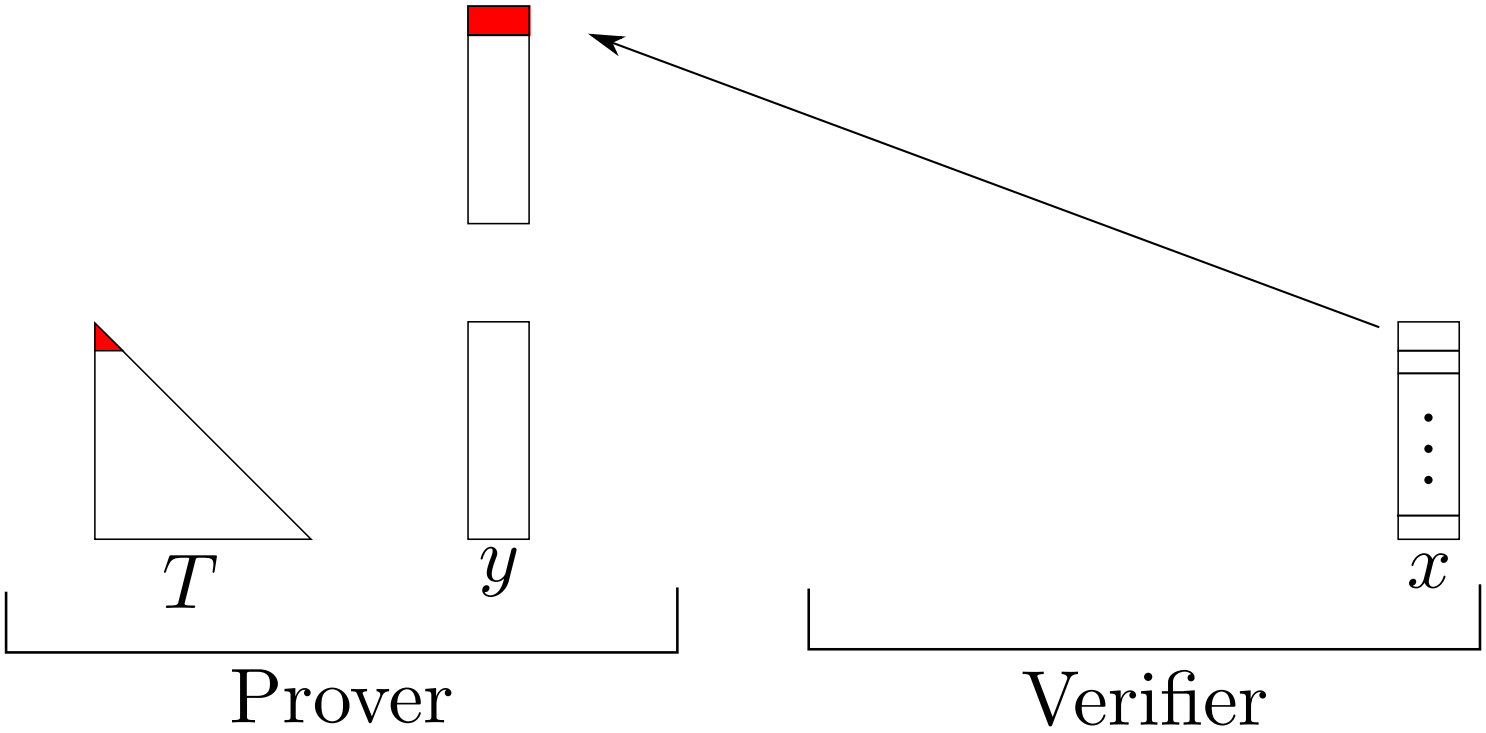

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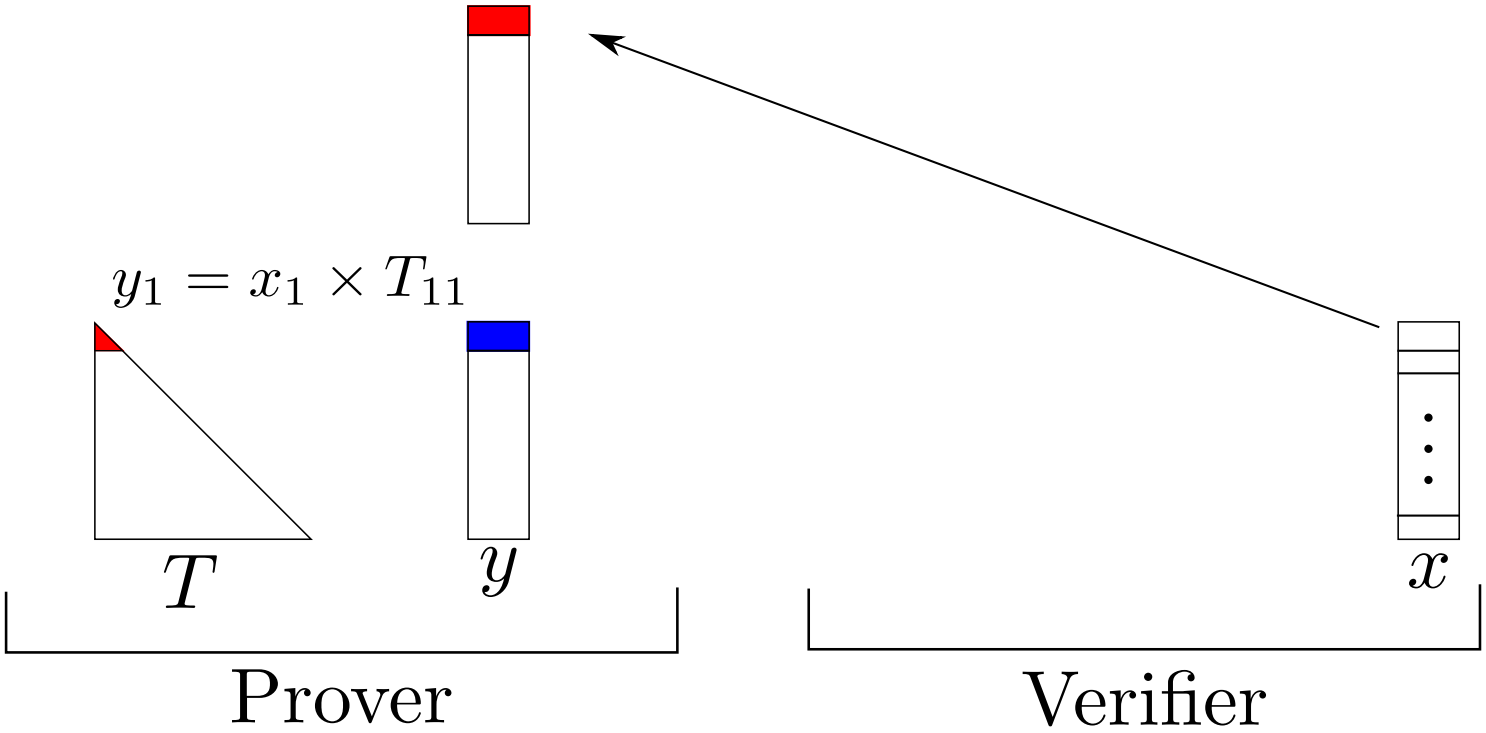

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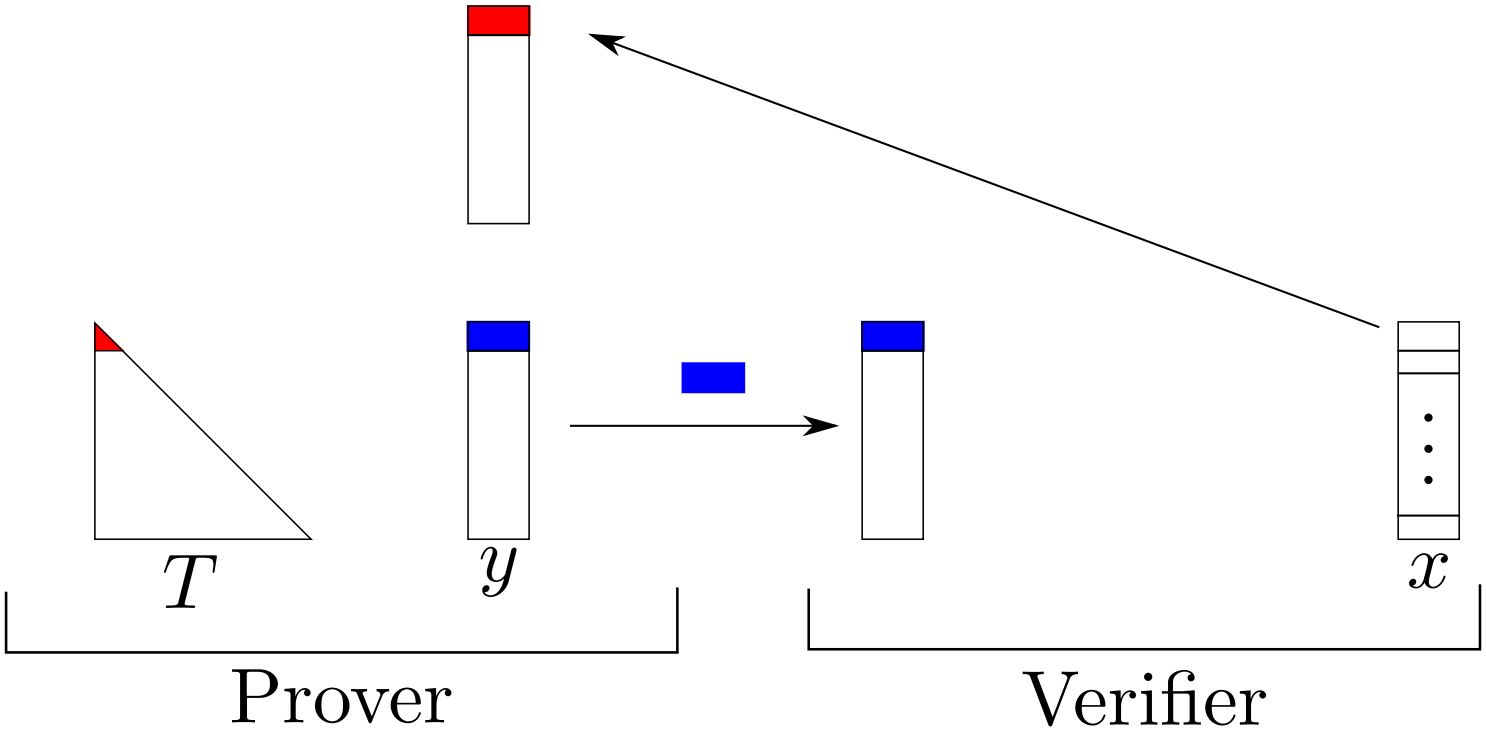

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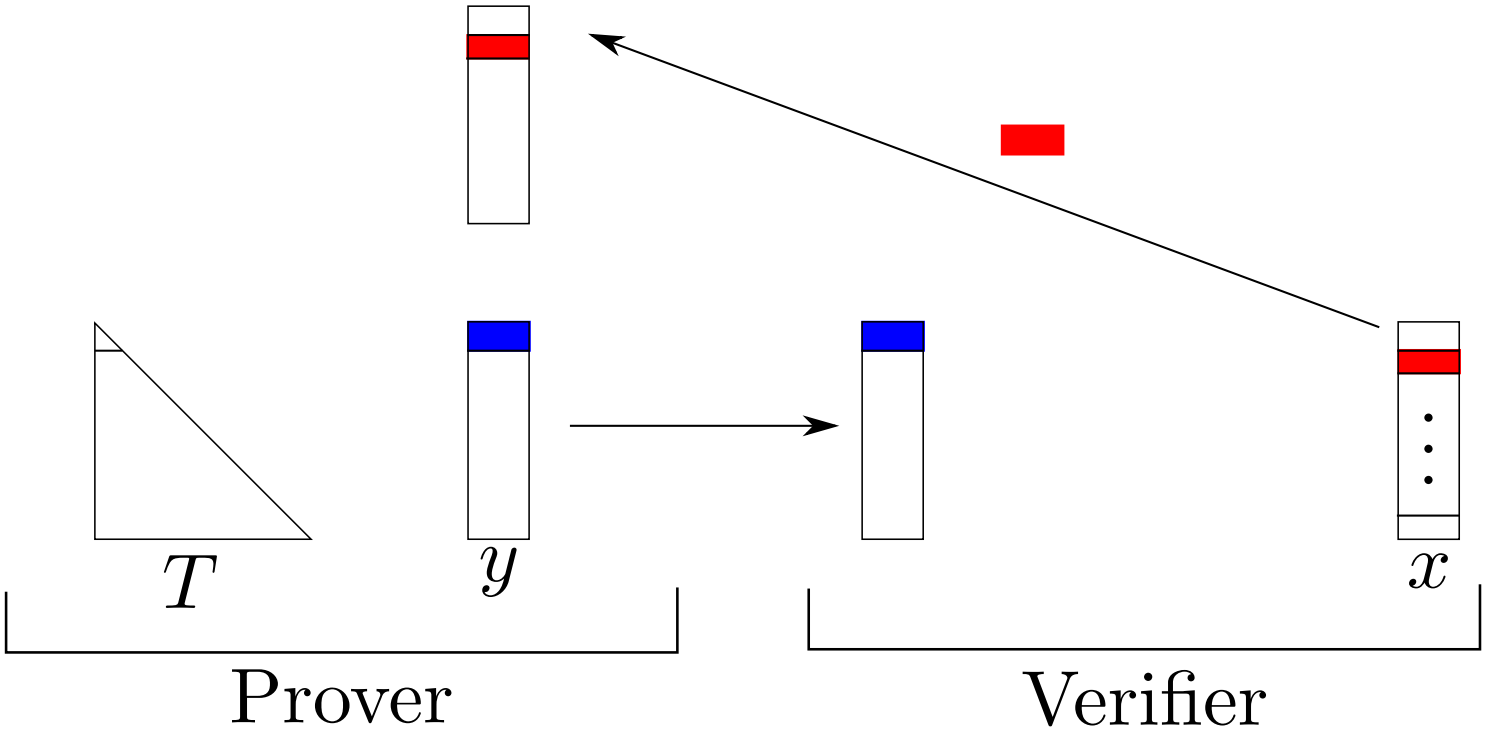

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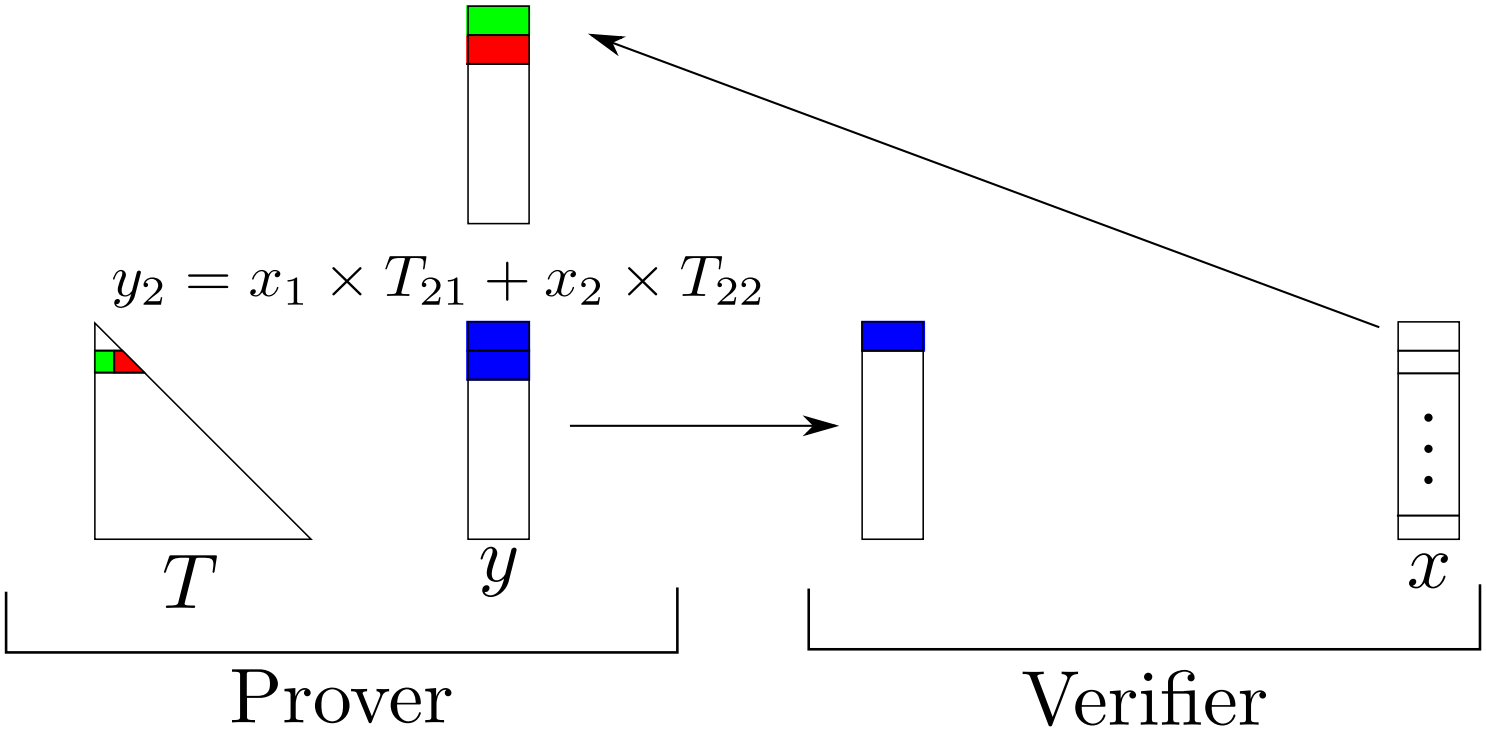

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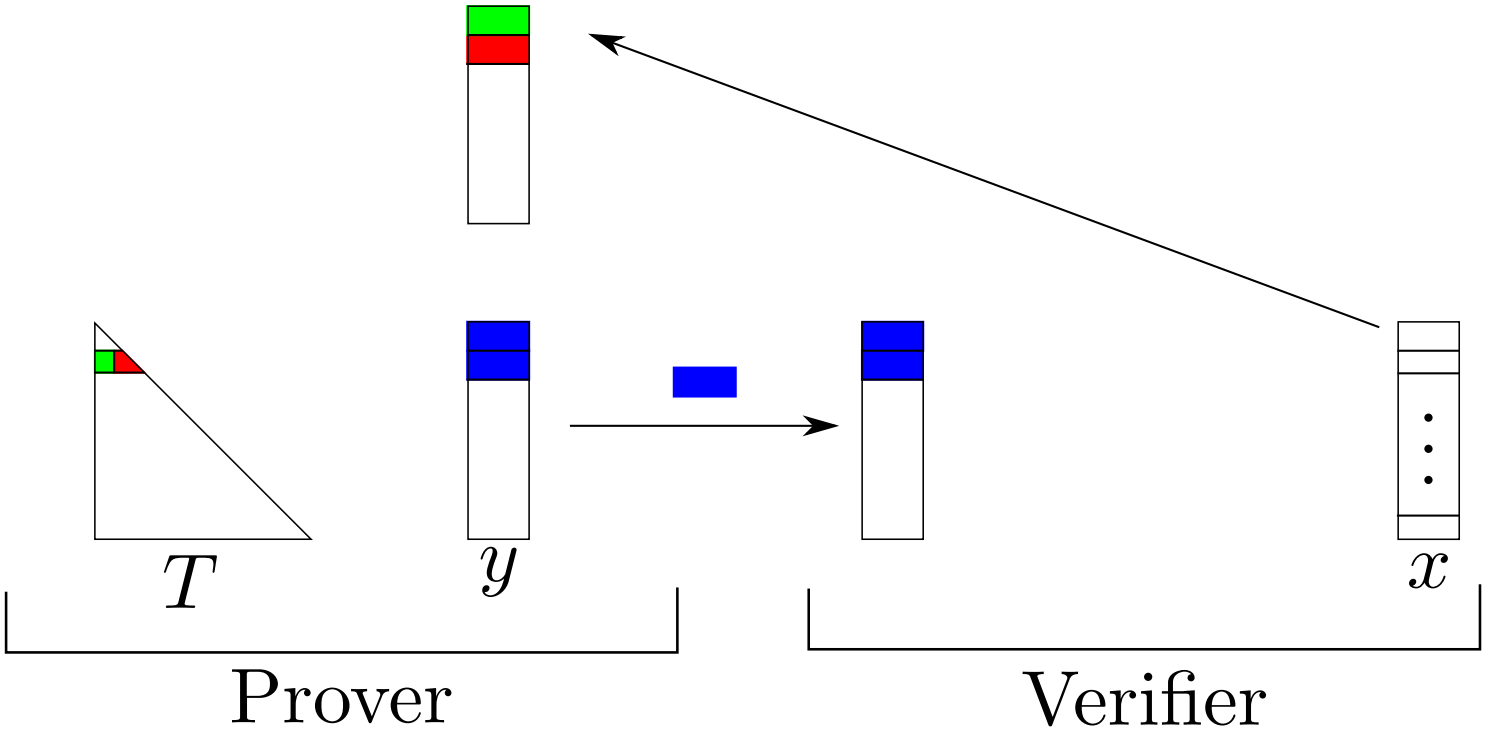

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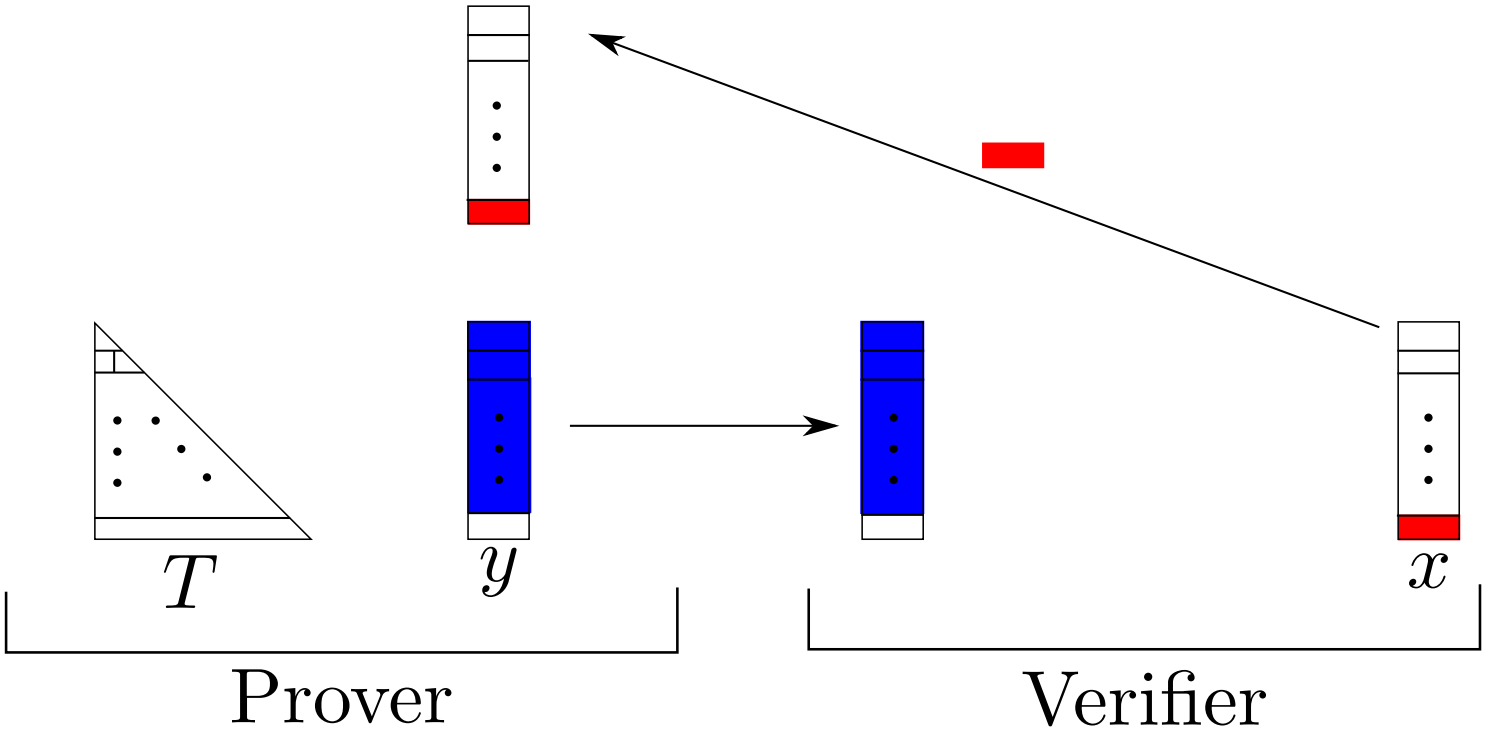

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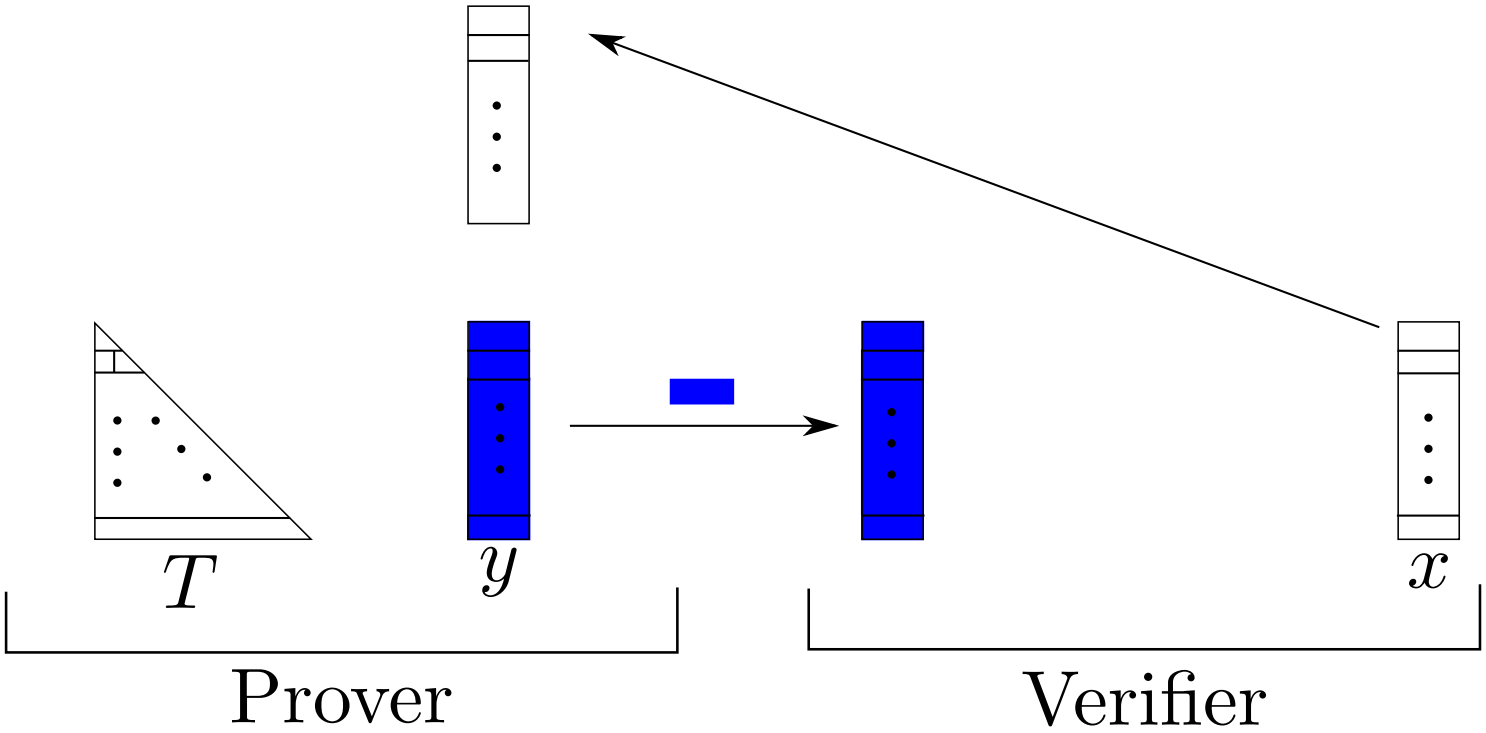

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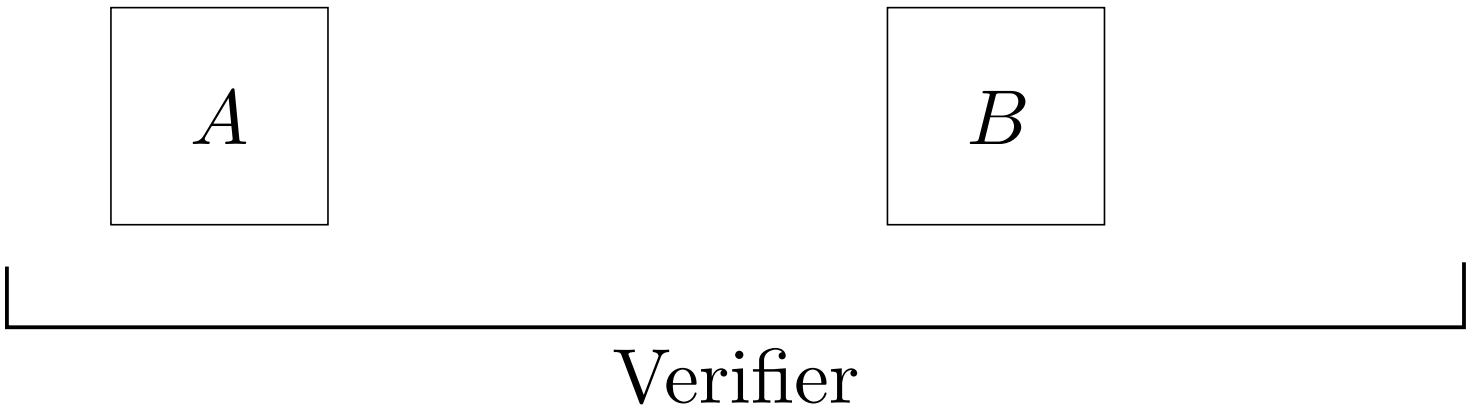

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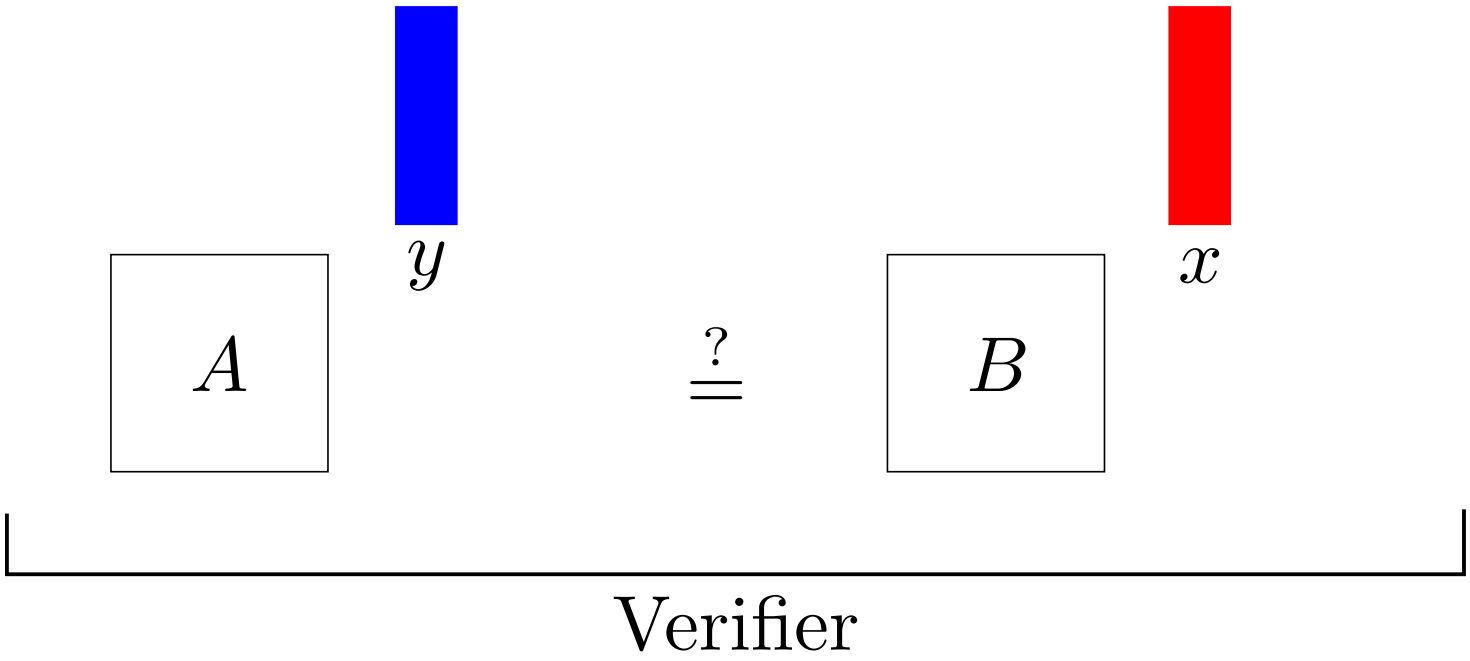

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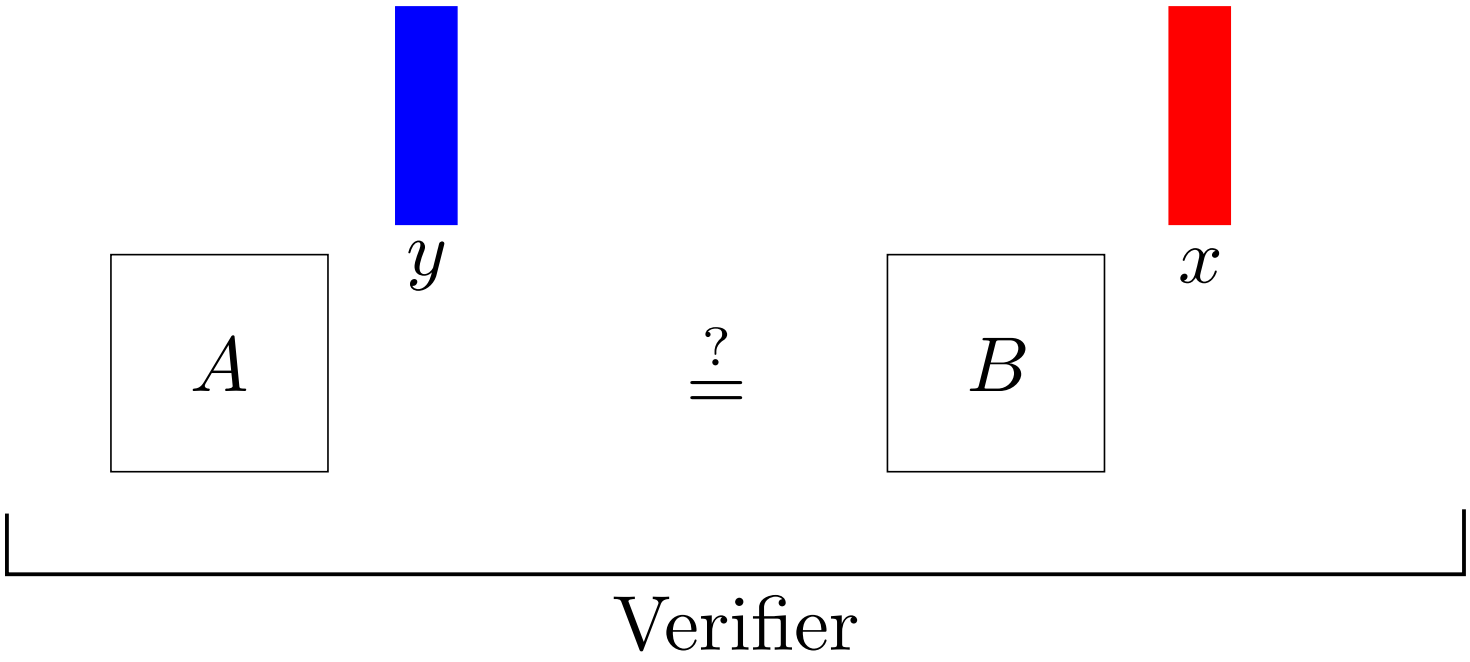
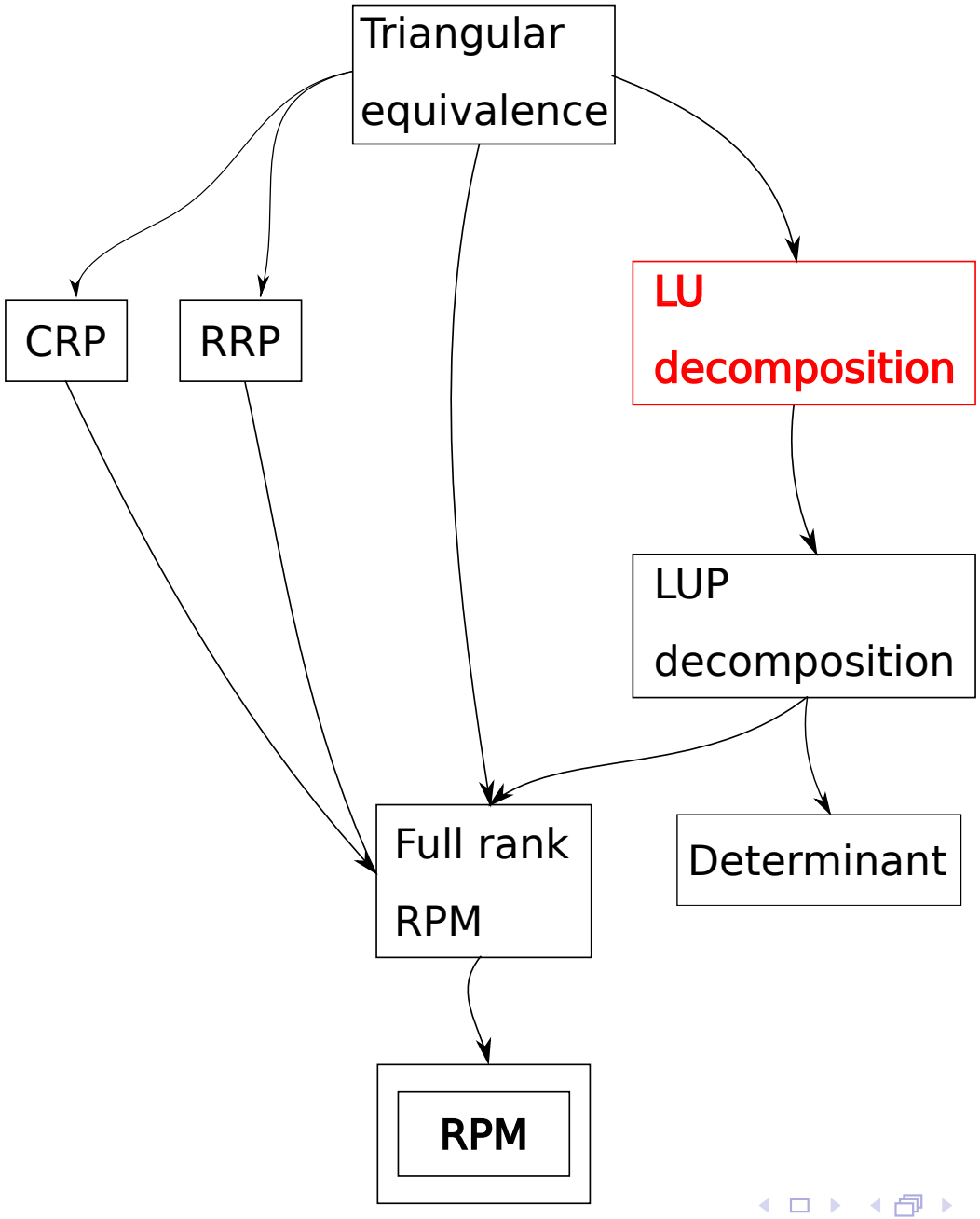


Prover	Comm.	Verifier
$O(mn^{\omega-1})$	$2n$	$O(n^2)$

Roadmap


Triangular equivalence
CRP
RRP
LU decomposition
LUP decomposition
Full rank RPM
Determinant
RPM

Certificate for LU decomposition

- Certificate for the **existence** of $A = LU$
- L and U are not transmitted
- Use the triangular equivalence scheme

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  - and $\lambda^T A \phi = \underbrace{\lambda^T L}_{z^T} \underbrace{U\phi}_{x}$

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- In that case, not enough!
- Not enough constraints

$\rightarrow$ Use two right projections instead

Certificate for LU decomposition

Prover		Verifier
	$A \in \mathbf{F}^{n \times n}$ non-singular	
$A = LU$	$\xrightarrow{A = LU}$	
	for i from n downto 1	
$\begin{bmatrix} x & y \end{bmatrix} = U \begin{bmatrix} \phi & \psi \end{bmatrix}$	$\xleftarrow{\phi_i, \psi_i}$ $\xrightarrow{x_i, y_i}$	(ϕ_i, ψ_i) random scalars
$z^T = \lambda^T L$	$\xleftarrow{\lambda_i}$ $\xrightarrow{z_i}$	λ_i random scalar
		$z^T \begin{bmatrix} x & y \end{bmatrix} \stackrel{?}{=} (\lambda^T A) \begin{bmatrix} \phi & \psi \end{bmatrix}$

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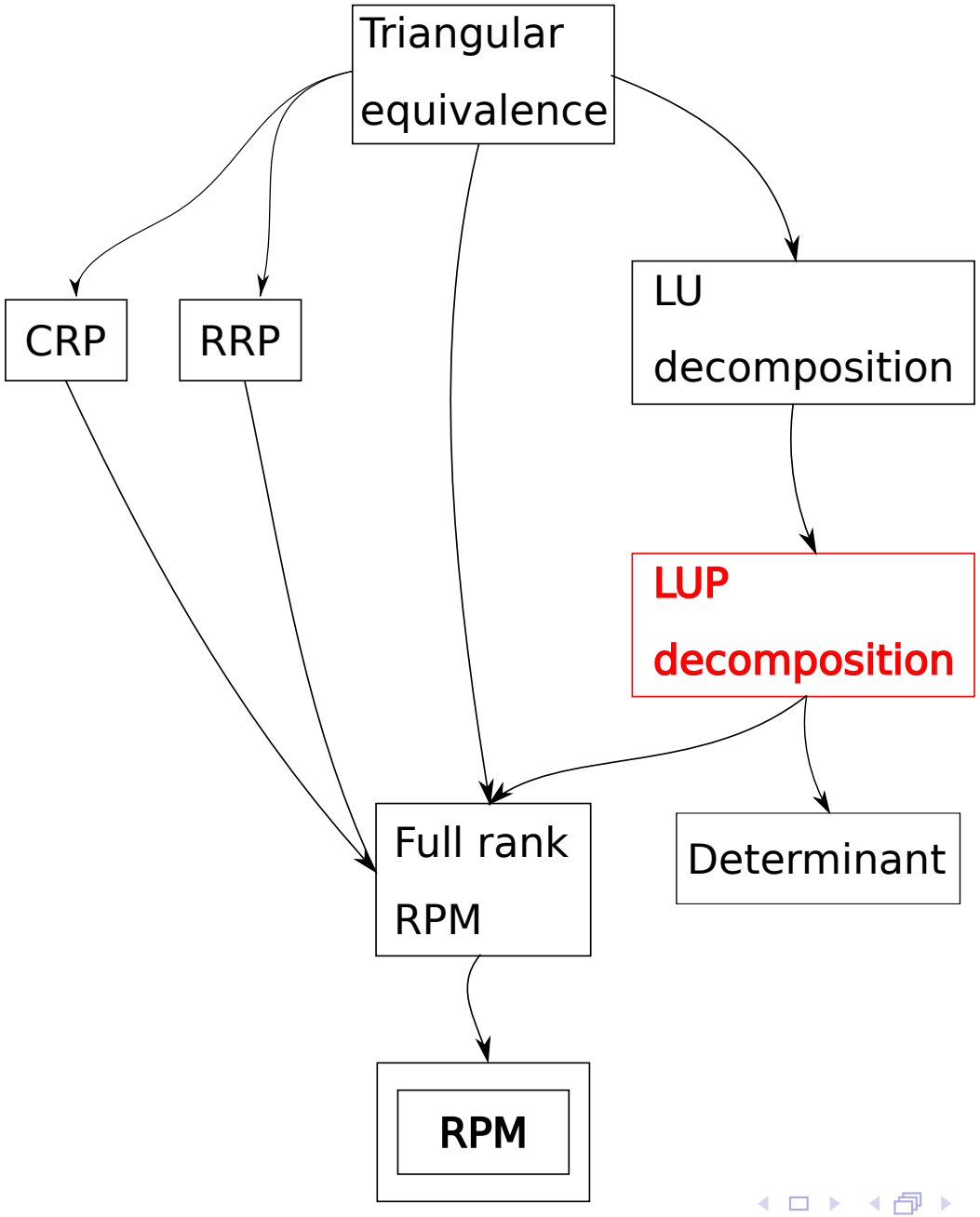
Prover	Comm.	Verifier
$O(n^\omega)$	$3n$	$O(n^2)$

Roadmap


Triangular equivalence
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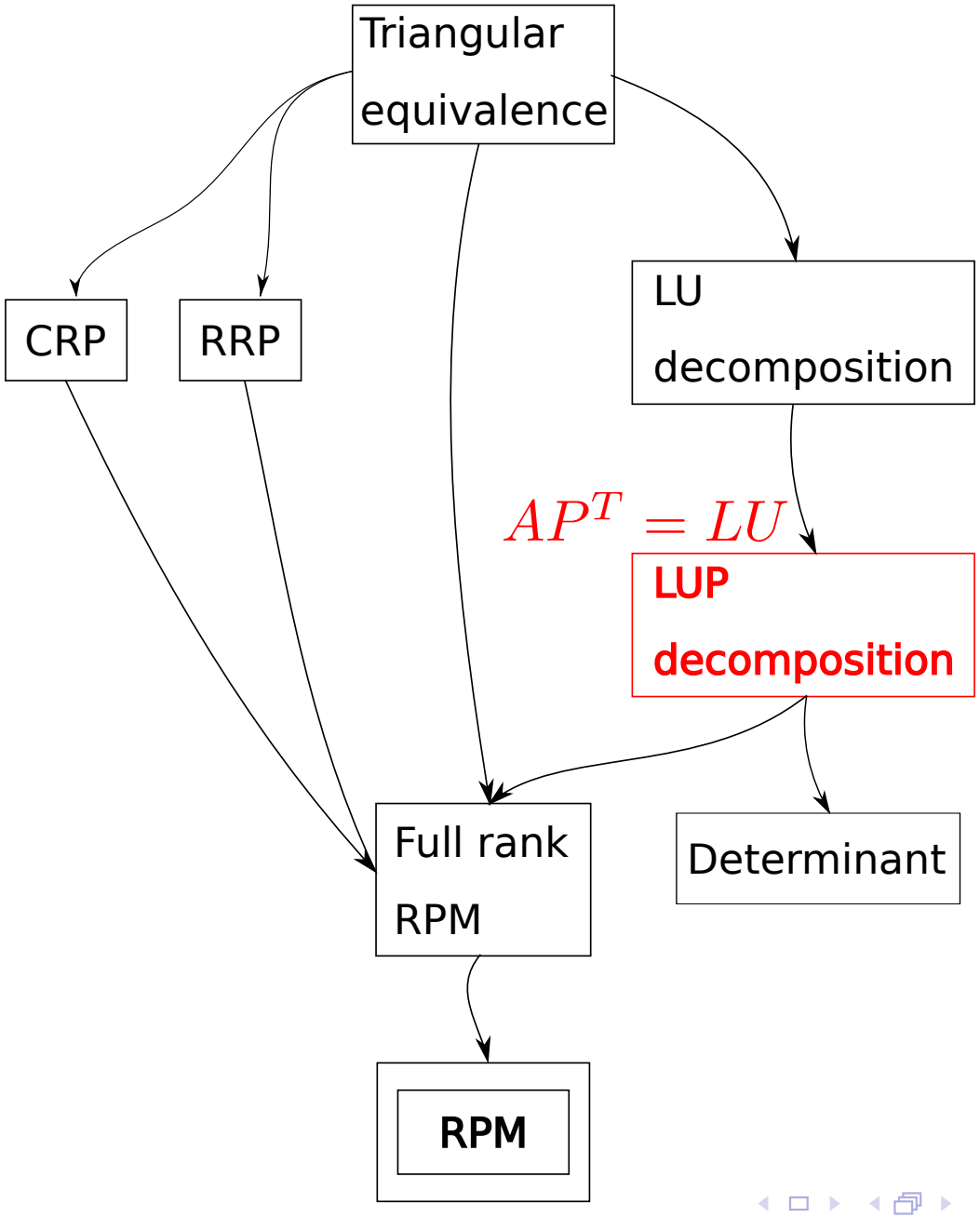
Triangular equivalence
CRP
RRP
LU decomposition
$AP^T = LU$
LUP decomposition
Full rank RPM
Determinant
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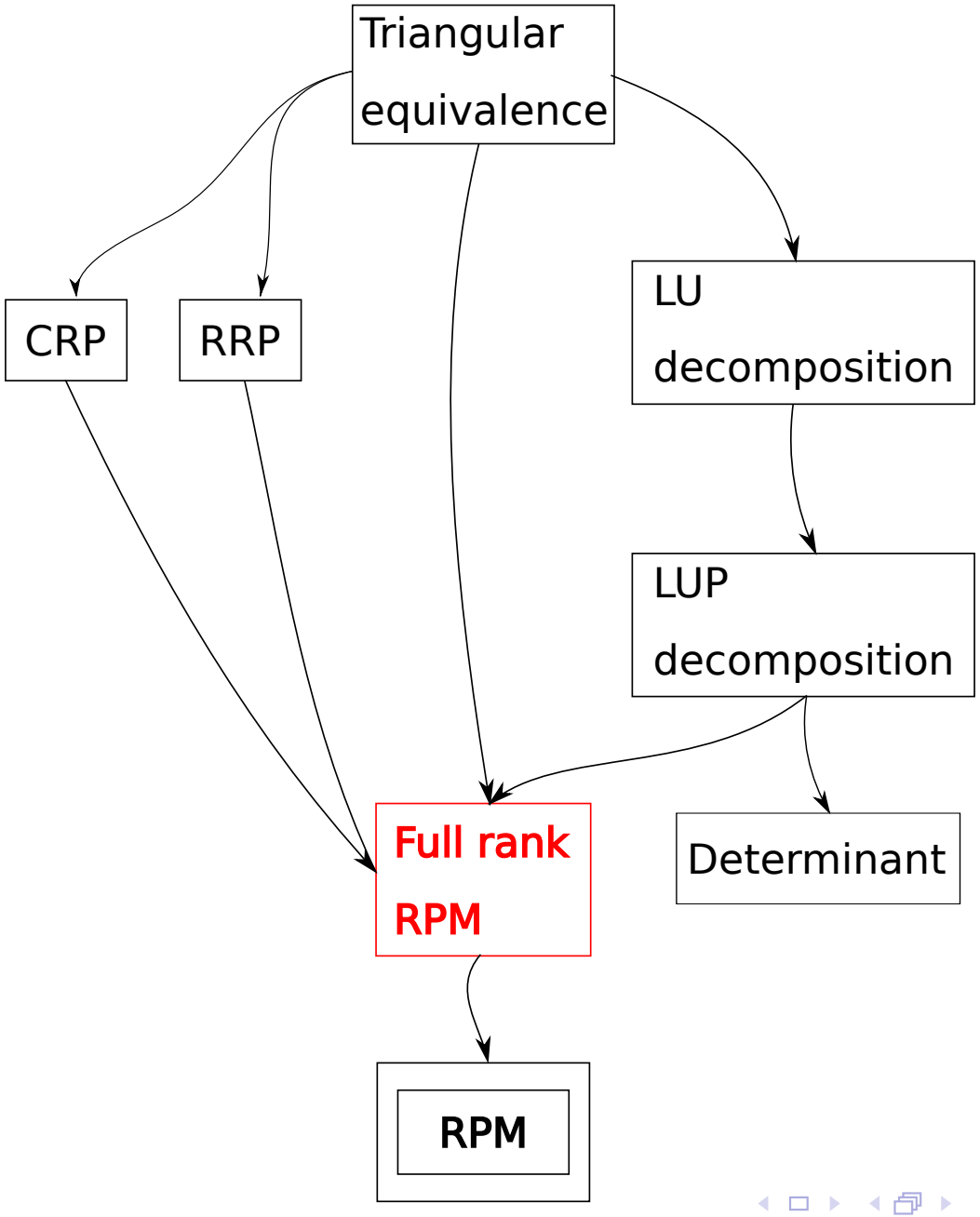
Roadmap


Triangular equivalence
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Rank profile matrix for an invertible matrix

- LUP decomposition that reveals the RPM $\iff \exists \overline{U} = P^T UP$
  - U and $\overline{U}$ upper triangular
- Difficulty: neither U nor $\overline{U}$ will be transmitted

Rank profile matrix for an invertible matrix

Prover | Verifier

$A \in \mathbf{F}^{n \times n}$ invertible

$A = LUP$

$\xrightarrow{P}$

$\xleftarrow{e}$

Triangular equivalence

$f^T \leftarrow e^T \overline{U}$

$\xrightarrow{f}$

$AP^T \stackrel{?}{=} LU$

$$\begin{bmatrix} x & y \end{bmatrix} = U \begin{bmatrix} \phi & \psi \end{bmatrix}$$

$$z^t = \lambda^T L$$

$\xrightarrow{z^T}$

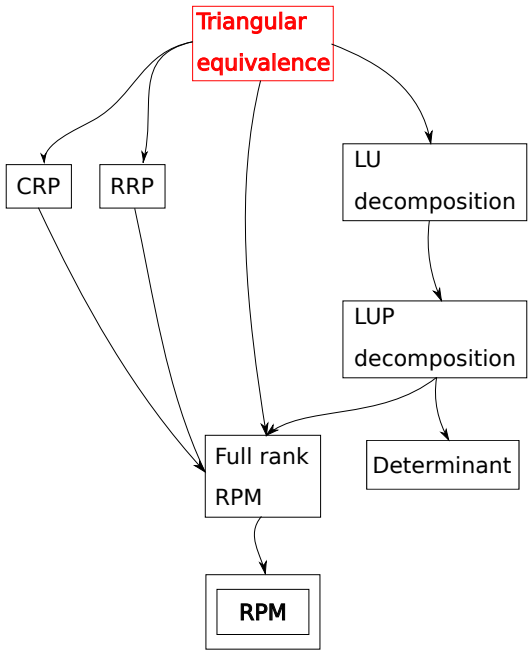
$\xrightarrow{x,y}$

$$z^T \begin{bmatrix} x & y \end{bmatrix} \stackrel{?}{=} \lambda^T A P^T \begin{bmatrix} \phi & \psi \end{bmatrix}$$

$e^T P^T x \stackrel{?}{=} f^T P^T \phi$

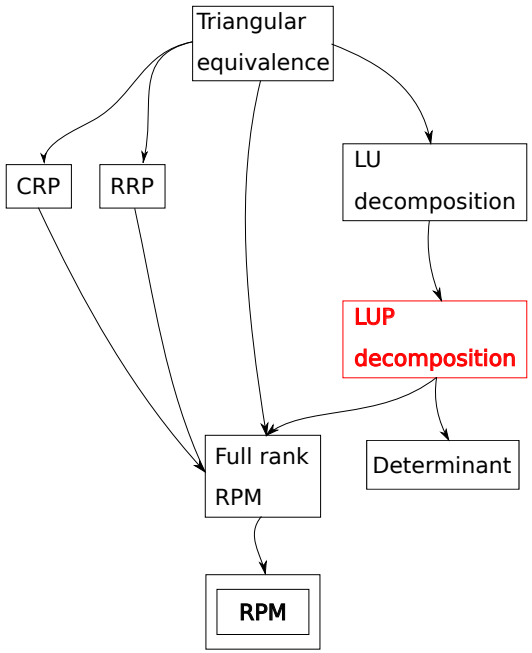
Summary

	Prover	Comm.	Verifier
Tri. eq.	Best known	$2n$	$O(n^2)$



Summary

	Prover	Comm.	Verifier
Tri. eq.	Best known	$2n$	$O(n^2)$
LUP	Best known	$8n$	$O(n^2)$



Summary

	Prover	Comm.	Verifier
Tri. eq.	Best known	$2n$	$O(n^2)$
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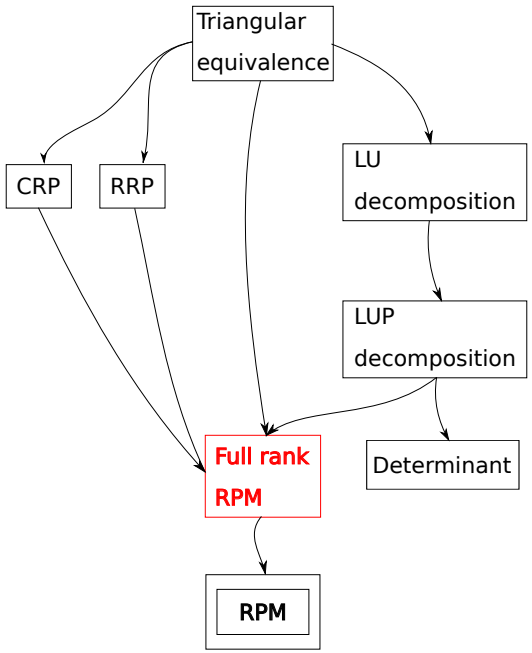
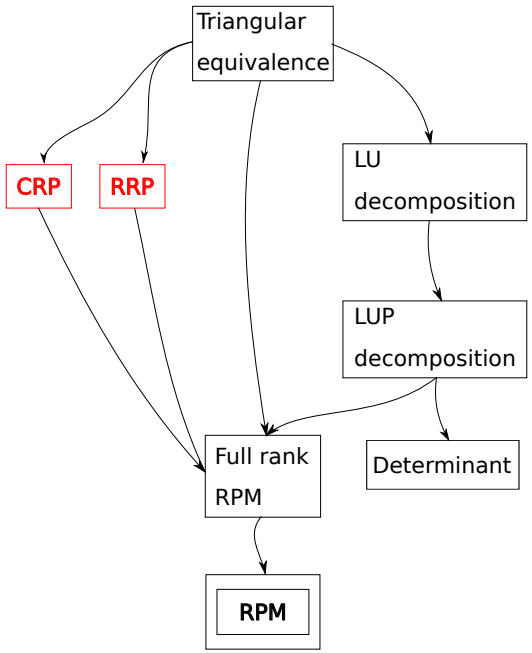
Full Rank RPM	Best known	$10n$	$O(n^2)$



Summary

	Prover	Comm.	Verifier
Tri. eq.	Best known	$2n$	$O(n^2)$
LUP	Best known	$8n$	$O(n^2)$
Full Rank RPM	Best known	$10n$	$O(n^2)$
CRP	Best known	$2n + 4r$	$O(n^2)$



Summary

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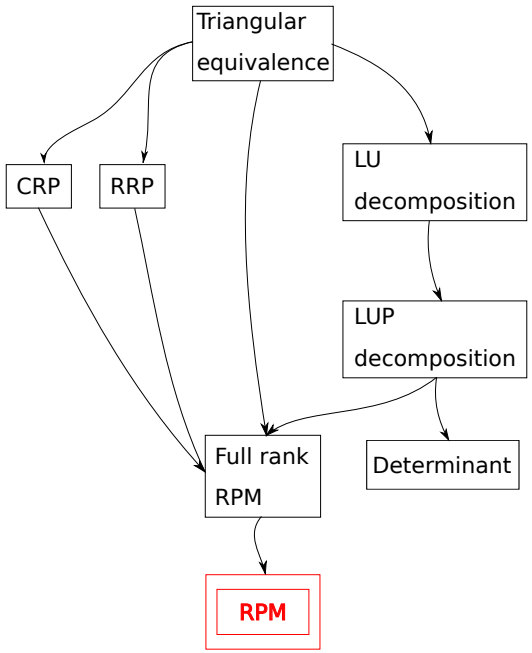
LUP	Best known	$8n$	$O(n^2)$
Full Rank RPM	Best known	$10n$	$O(n^2)$
CRP	Best known	$2n + 4r$	$O(n^2)$
RPM	Best known	$3n + 16r$	$O(n^2)$



Plan

1. Introduction
2. Our certificates
3. Conclusion

Conclusion

- Application:
  - Dense determinant certificate from LUP instead of `CHARPOLY`
  - About 5 times more efficient for the prover than the dense case of [DKTV16]
- Perspectives:
  - LDL^T certificate and symmetric positive-definiteness

Determinant computation

	Prover	Comm.	Verifier
[F79]	Best known dense	$O(n^2)$	$O(n^2)$
[DKTV16]	Best known sparse	$5n$	$O(n^2)$
This paper	Best known dense	$8n$	$O(n^2)$

- Computation time measured using FFLAS-FFPACK
- Single core Intel Skylake @3.4GHz
- Communication over Ethernet Cat. 6, @1Gb/s cable
- 64 bit words, random matrices modulo 131071

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Dimension	$2k$	$10k$	$50k$
PLUQ	0.28s	17.99s	1448.16s
CharPoly	1.96s	100.37s	8047.56s
Linear comm.	0.50s	0.50s	0.50s
Quadratic comm.	1.50s	7.50s	222.68s
`fgemv`	0.0013s	0.038s	1.03s